APPENDIX 21A
Example: Development of a Principal Spillway Mass Curve

The Principal Spillway Hydrograph (PSH) is one of the design hydrographs required for analyzing a dam's design to check that it meets NRCS design criteria. This example problem illustrates the development of the mass curve for the principal spillway design storm event for a dam on a watershed with the characteristics as described below. This example stops with development of the mass curve. In doing an actual analysis, the mass curve developed using the procedure illustrated here would be combined with the watershed's unit hydrograph to develop a hydrograph called the Principal Spillway Hydrograph or Principal Spillway Design Hydrograph.

Watershed Characteristics:
Drainage Area, DA = 15.0 square miles
Time of concentration, T_c = 7.1 hours
Average annual precipitation, P_a = 22.8 inches
Average annual temperature, T_a = 61.5° F
Runoff curve number, CN = 80
100-year, 1-day precipitation,
$P_{100,1\text{-day}}$ = 6.8 inches
100-year, 10-day precipitation,
$P_{100,10\text{-day}}$ = 11.0 inches

The principal spillway design storm for this watershed is the 100-year return interval storm.

Step 1: Determine the areal rainfall.
Adjustment factors are found in Table 21-2.

For a drainage area of 15.0 square miles, the 1-day precipitation adjustment factor is 0.978 and the 10-day precipitation adjustment factor is 0.991.

Adjusted rainfalls are:
$P_{100,1\text{-day adjusted}} = 0.978(6.9)$
$= 6.65$ inches
$P_{100,10\text{-day adjusted}} = 0.991(11.0)$
$= 10.9$ inches

Step 2: Determine the Curve Number for the10-day precipitation.
The 100-year frequency 10-day precipitation amount is greater that 6 inches, thus table 21-3 is used to determine the 10-day curve number.

For $CN_{1\text{-day}} = 80$, from table 21-3, the $CN_{10\text{-day}} = 65$

See the footnote for the table to determine when table 21-3 applies.

Step 3: Estimate the direct runoff for 1- and 10-days.
Use the runoff equation or use Appendix 10A (NEH-630, Chapter 10, Estimation of Direct Runoff from Storm Rainfall) to determine the direct runoff for the 1-day and 1-day events.

1-day Runoff:

$$Q_{1-day} = \frac{\left[P_{1day} - 0.2 \times \left(\frac{1000}{CN_{1-day}} - 10\right)\right]^2}{\left[P_{1day} + 0.8 \times \left(\frac{1000}{CN_{1-day}} - 10\right)\right]}$$

$$Q_{1-day} = \frac{\left[6.65 - 0.2 \times \left(\frac{1000}{80} - 10\right)\right]^2}{\left[6.65 + 0.8 \times \left(\frac{1000}{80} - 10\right)\right]}$$

$Q_{1\text{-}day}$ = 4.37 inches

10-day Runoff:

$$Q_{10-day} = \frac{\left[P_{10day} - 0.2 \times \left(\frac{1000}{CN_{10-day}} - 10\right)\right]^2}{\left[P_{10day} + 0.8 \times \left(\frac{1000}{CN_{10-day}} - 10\right)\right]}$$

$$Q_{10-day} = \frac{\left[10.9 - 0.2 \times \left(\frac{1000}{65} - 10\right)\right]^2}{\left[10.9 + 0.8 \times \left(\frac{1000}{65} - 10\right)\right]}$$

$Q_{10\text{-}day}$ = 6.34 inches

Step 4: Compute the climatic index.

Using the given data and equation 21-1 determine C_i.

$$C_i = \frac{100 \times P_a}{(T_a)^2}$$

$$C_i = \frac{100 \times 22.8}{(61.5)^2}$$

$C_i = 0.603$

Because the C_i is less than 1 the channel loss may be used to reduce direct runoff.

Step 5: Estimate the net runoff.

The net runoff is the direct runoff multiplied by the channel loss reduction factor determined from table 21.4.

Enter table 21.4 with the drainage area of 15.0 square miles and the C_i of 0.603 and by interpolation find a reduction factor of 0.75.

Multiply the runoff volumes by the channel loss reduction factor to get the net runoff volumes which will be used for the rest of the example.

$Q_{1\text{-}day\text{-}net} = Q_{1\text{-}day}$ x Reduction Factor

$Q_{1\text{-}day\text{-}net}$ = 4.37 x 0.75
$Q_{1\text{-}day\text{-}net}$ = 3.28 inches

$Q_{10\text{-}day\text{-}net}$ = $Q_{10\text{-}day}$ x Reduction Factor
$Q_{10\text{-}day\text{-}net}$ = 6.34 x 0.75
$Q_{10\text{-}day\text{-}net}$ = 4.76 inches

Step 6: Determine the mass curve of runoff.

Equation 21-2 gives the distribution for developing the mass curve. The equation in its ready form is

$$Q_D = Q_{10-day-net}\left(\frac{D}{10}\right)^a$$

where: Q_D = total runoff at time D, days
$Q_{10\text{-}day\text{-}net}$ = net runoff at the end of 10 days = 4.76 inches
a = log ($Q_{10\text{-}day\ net}/Q_{1\text{-}day\ net}$)
$Q_{1\text{-}day\text{-}net}$ = net runoff at end of 1 day = 3.28 inches
a = log (4.76/3/28) = 0.1617

Substituting gives:

$$Q_D = 4.76\left(\frac{D}{10}\right)^{0.1617}$$

Step 7: Calculate the mass curve.

a) The equation from Step 6 above is used to calculate the mass curve. The curve is tabulated on 0.1-day, or 2.4 hour, time increments, as shown in Table 21A-1.
b) The calculated runoff volumes are shown in column (a) in Table 21A-1, labeled Q_D.
c) Incremental volumes are tabulated on a 0.1 day time increment in column (b), labeled $\Delta QD_{0.1\text{-}day}$, in Table 21A-1.
d) Incremental volumes are also tabulated on a 0.5 day time increment in column (c), labeled $\Delta QD_{0.5\text{-}day}$, in Table 21A-1. (Note that both the 0.1-day and 0.5-day tabulations are needed in developing the reordered mass curve described as follows.)

Table 21A-1. Calculated runoff volumes, Q_D and Incremental Runoff Volumes, ΔQ_D for 0.1 and 0.5 day increments.

Time: (days)	(a) Qd: (inches)	(b) $\Delta QD_{0.1\text{-day}}$ (inches)	(c) $\Delta QD_{0.5\text{-day}}$ (inches)	Time: (days)	(a) Qd: (inches)	(b) $\Delta QD_{0.1\text{-day}}$ (inches)	(c) $\Delta QD_{0.5\text{-day}}$ (inches)
0	0.000						
0.1	2.260	2.2602		**5.1**	4.269	0.0137	
0.2	2.528	0.2681		**5.2**	4.282	0.0134	
0.3	2.700	0.1714		**5.3**	4.295	0.0132	
0.4	2.828	0.1286		**5.4**	4.309	0.0130	
0.5	2.932	0.1039	2.9322	**5.5**	4.321	0.0128	0.0661
0.6	3.020	0.0877		**5.6**	4.334	0.0126	
0.7	3.096	0.0762		**5.7**	4.346	0.0124	
0.8	3.164	0.0676		**5.8**	4.359	0.0122	
0.9	3.225	0.0608		**5.9**	4.371	0.0121	
1	3.280	0.0554	0.3478	**6**	4.383	0.0119	0.0612
1.1	3.331	0.0510		**6.1**	4.394	0.0117	
1.2	3.378	0.0472		**6.2**	4.406	0.0116	
1.3	3.422	0.0440		**6.3**	4.417	0.0114	
1.4	3.463	0.0413		**6.4**	4.429	0.0113	
1.5	3.502	0.0389	0.2223	**6.5**	4.440	0.0111	0.0571
1.6	3.539	0.0367		**6.6**	4.451	0.0110	
1.7	3.574	0.0349		**6.7**	4.461	0.0108	
1.8	3.607	0.0332		**6.8**	4.472	0.0107	
1.9	3.639	0.0317		**6.9**	4.483	0.0106	
2	3.669	0.0303	0.1668	**7**	4.493	0.0104	0.0535
2.1	3.698	0.0291		**7.1**	4.504	0.0103	
2.2	3.726	0.0279		**7.2**	4.514	0.0102	
2.3	3.753	0.0269		**7.3**	4.524	0.0101	
2.4	3.779	0.0259		**7.4**	4.534	0.0100	
2.5	3.804	0.0250	0.1348	**7.5**	4.544	0.0099	0.0504
2.6	3.828	0.0242		**7.6**	4.553	0.0097	
2.7	3.852	0.0234		**7.7**	4.563	0.0096	
2.8	3.874	0.0227		**7.8**	4.573	0.0095	
2.9	3.896	0.0221		**7.9**	4.582	0.0094	
3	3.918	0.0214	0.1138	**8**	4.591	0.0093	0.0477
3.1	3.939	0.0208		**8.1**	4.601	0.0092	
3.2	3.959	0.0203		**8.2**	4.610	0.0091	
3.3	3.979	0.0198		**8.3**	4.619	0.0090	
3.4	3.998	0.0193		**8.4**	4.628	0.0090	
3.5	4.017	0.0188	0.0989	**8.5**	4.637	0.0089	0.0452
3.6	4.035	0.0183		**8.6**	4.645	0.0088	
3.7	4.053	0.0179		**8.7**	4.654	0.0087	
3.8	4.070	0.0175		**8.8**	4.663	0.0086	
3.9	4.088	0.0171		**8.9**	4.671	0.0085	
4	4.104	0.0168	0.0877	**9**	4.680	0.0084	0.0431
4.1	4.121	0.0164		**9.1**	4.688	0.0084	
4.2	4.137	0.0161		**9.2**	4.696	0.0083	
4.3	4.153	0.0158		**9.3**	4.704	0.0082	
4.4	4.168	0.0155		**9.4**	4.713	0.0081	
4.5	4.183	0.0152	0.0789	**9.5**	4.721	0.0081	0.0411
4.6	4.198	0.0149		**9.6**	4.729	0.0080	
4.7	4.213	0.0146		**9.7**	4.737	0.0079	
4.8	4.227	0.0144		**9.8**	4.744	0.0079	
4.9	4.241	0.0141		**9.9**	4.752	0.0078	
5	4.255	0.0139	0.0719	**10**	4.760	0.0077	0.0393

e) As described in Table 21-6, reorder the mass curve incremental volumes on the time intervals described (a combination of 0.1-day and 0.5-day increments). This order for the arrangement of increments per Table 21-6 is shown in column (d) below. The reordered incremental volumes are shown in column (e), labeled “Reordered Interval Runoff”."

f) Reaccumulate the interval volumes. Shown in column (f), labeled "Accumulated PSMC" in table 21A-2.

g) Divide each of the values in column (e) by the total net runoff volume to convert to a dimensionless mass curve. Shown in column (g), labeled "Dimensionless PSMC."

Table 21A-2. Principal Spillway Mass Curve.

Time (days)	*(d) Increment arrangement (per Table 21-6):*	*(e) Reordered Interval Runoff:*	*(f) Accumulated PSMC:*	*(g)* ***Dimensionless PSMC:***
0.0			*0*	**0.0000**
0.5	*19th largest 1/2 day*	0.0411	*0.04*	**0.0086**
1.0	*17th largest 1/2 day*	0.0452	*0.09*	**0.0181**
1.5	*15th largest 1/2 day*	0.0504	*0.14*	**0.0287**
2.0	*13th largest 1/2 day*	0.0571	*0.19*	**0.0407**
2.5	*11th largest 1/2 day*	0.0661	*0.26*	**0.0546**
3.0	*9th largest 1/2 day*	0.0789	*0.34*	**0.0712**
3.5	*7th largest 1/2 day*	0.0989	*0.44*	**0.0920**
4.0	*5th largest 1/2 day*	0.1348	*0.57*	**0.1203**
4.5	*3rd largest 1/2 day*	0.2223	*0.79*	**0.1670**
4.6	*9th largest 1/10 day*	0.0608	*0.86*	**0.1798**
4.7	*7th largest 1/10 day*	0.0762	*0.93*	**0.1958**
4.8	*5th largest 1/10 day*	0.1039	*1.04*	**0.2176**
4.9	*3rd largest 1/10 day*	0.1714	*1.21*	**0.2536**
5.0	*Largest 1/10 day*	2.2602	*3.47*	**0.7285**
5.1	*2nd largest 1/10 day*	0.2681	*3.74*	**0.7848**
5.2	*4th largest 1/10 day*	0.1286	*3.86*	**0.8118**
5.3	*6th largest 1/10 day*	0.0877	*3.95*	**0.8302**
5.4	*8th largest 1/10 day*	0.0676	*4.02*	**0.8444**
5.5	*10th largest 1/10 day*	0.0554	*4.07*	**0.8561**
6.0	*4th largest 1/2 day*	0.1668	*4.24*	**0.8911**
6.5	*6th largest 1/2 day*	0.1138	*4.36*	**0.9150**
7.0	*8th largest 1/2 day*	0.0877	*4.44*	**0.9335**
7.5	*10th largest 1/2 day*	0.0719	*4.52*	**0.9486**
8.0	*12th largest 1/2 day*	0.0612	*4.58*	**0.9614**
8.5	*14th largest 1/2 day*	0.0535	*4.63*	**0.9727**
9.0	*16th largest 1/2 day*	0.0477	*4.68*	**0.9827**
9.5	*18th largest 1/2 day*	0.0431	*4.72*	**0.9917**
10.0	*20th largest 1/2 day*	0.0393	*4.76*	**1.0000**

8. Plotting time versus the dimensionless values results in the following graph of the principal spillway mass curve, similar to curve C shown in Figure 21-7.

Note: Once the principal spillway mass curve is developed, the procedures in Chapter 16 are followed to develop the actual principal spillway hydrograph. The example found in Appendix B also illustrates computation of the actual hydrograph from the mass curve for a freeboard storm. It is important to remember that the time increment of both the mass curve and unit hydrograph must be the same.

Figure 21A-1: Principal Spillway Mass Curve of Runoff