APPENDIX 21B
Example: Development of a Design Hydrograph: Freeboard Hydrograph (FBH) Storm

In order to check the capacity of the earthen spillway of a dam installed on the watershed described in Appendix 21A, the 24-hour freeboard hydrograph must be developed.

To recap from the example in Appendix 21A, the watershed characteristics are as follows:

Drainage area = 15.0 square miles
Time of concentration = 7.1 hours
Runoff Curve Number = 80
Structure Hazard Class: High hazard

Development of the freeboard hydrograph for this example will use the 5-point distribution as described on page 21-14.

STEP 1: Determine the appropriate time interval to use.
Both the unit hydrograph and rainfall distribution must be compiled on the same time interval for developing the final runoff hydrograph.

Using Chapter 16, Equation 16A-13, $\Delta D = 0.133Tc$ or $0.133(7.1) = 0.9443$ hour. For convenience, round this to $\Delta D = 1$ hour.

.
STEP 2: Develop the rainfall temporal distribution
a. Determine the probable maximum precipitation (PMP) amounts using the appropriate hydrometeorological report (HMR) or other appropriate special study. For this example, the PMP values are as follows:
$PMP_{6\text{-hour duration}}$: 29.0 inches
$PMP_{12\text{-hour duration}}$: 34.0 inches
$PMP_{24\text{-hour duration}}$: 38.0 inches

b. Distribute the rainfall into four 6-hour increments and determine the fraction of rainfall for each increment as described on page 21-15 and shown in Table 21B-1.

Table 21B-1: Distribution of rainfall into time increments for the 5-point rainfall distribution

Time increment (hours):	Incremental Rainfall Volume:	Incremental Rainfall Volume (inches):	Rainfall Fractions:
0 to 6	$0.5 \times (PMP_{24\text{-hour}} - PMP_{12\text{-hour}}) = 0.5 \times (38.0\text{-}34.0) =$	2.0	0.053
6 to 12	$PMP_{6\text{-hour}} =$	29.0	0.763
12 to 18	$PMP_{12\text{-hour}} - PMP_{6\text{-hour}} = 34.0 - 29.0 =$	5.0	0.132
18 to 24	$0.5 \times (PMP_{24\text{-hour}} - PMP_{12\text{-hour}}) = 0.5 \times (38.0\text{-}34.0) =$	2.0	0.053

c. Accumulate the rainfall fractions for each time increment to develop the final 5-point rainfall distribution as shown in Table 21B-2.

Table 21B-2: 5-point Rainfall Distribution for 6-hour increments

Time Ending	Rainfall Fraction:	5-point Rainfall

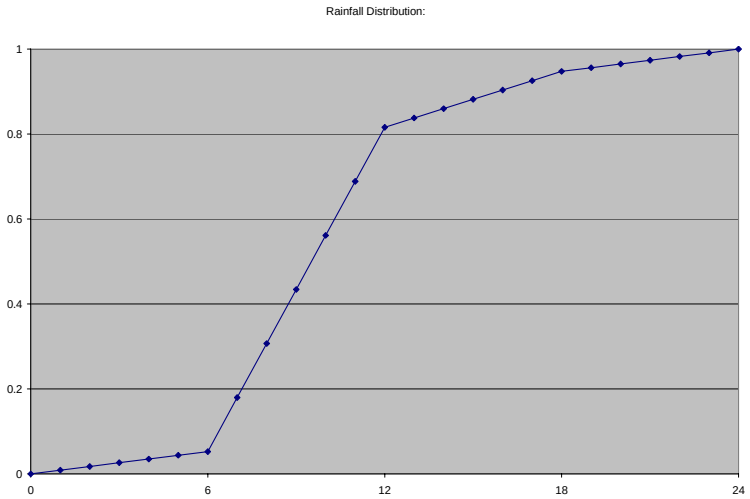
(hour):		Distribution:
0		0
6	0.053	0.053
12	0.763	0.816
18	0.132	0.947
24	0.053	1.000

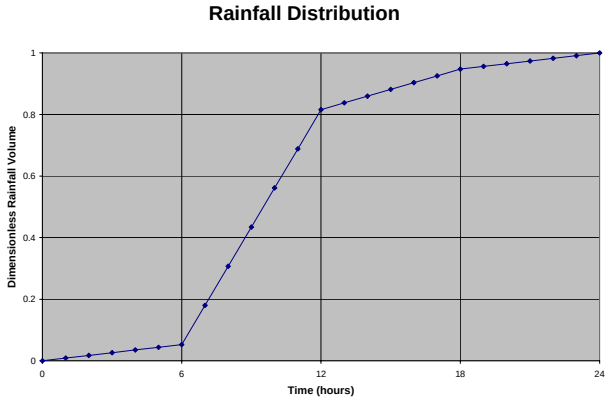
d. Reaccumulate the rainfall distribution on the time interval, ΔD = 1 hour, determined in Step 1. Table 21B-3 shows the rainfall distribution from Table 21B-2 reaccumulated on an hourly basis. Figure 21B-1 is a plot of the rainfall distribution from Table 21B-3.

Table 21B-3: Rainfall Distribution for ΔD = 1 hour time increments

Time (hours):	Rainfall Distribution:
0	0
1	*0.009*
2	*0.018*
3	*0.026*
4	*0.035*
5	*0.044*
6	0.053
7	*0.180*
8	*0.307*
9	*0.434*
10	*0.561*
11	*0.689*
12	0.816
13	*0.838*
14	*0.860*
15	*0.882*
16	*0.904*
17	*0.925*
18	0.947
19	*0.956*
20	*0.965*
21	*0.974*
22	*0.982*
23	*0.991*
24	1.000

Figure 21B-1: Plotted Rainfall Distribution





STEP 3: Develop the mass curve of runoff for the design storm.
This follows the procedures outline in NEH-630.16.

Starting with the Rainfall Distribution accumulated on 1-hour time increments from Step 2, determine the incremental runoff as shown in Table 21B-6.
- Columns (a) and (b) show the hourly rainfall distribution as tabulated in step 2.
- Column (c) shows the total accumulated rainfall distribution determined by multiplying the values in column (b) by $PMP_{24\text{-hour}}$ of 38.0 inches.
- Column (d) shows the accumulated runoff volume determined using the Curve Number Runoff equation (see NEH-630.10) with an RCN = 80 (assuming Antecedent Runoff Condition II).
- Column (e) shows the incremental mass curve of runoff volume for each 1-hour time increment.
Note that for RCN=80, runoff does not begin until rainfall is greater than the initial abstraction of 0.2S = 0.2 (2.5) = 0.5.

Table 21B-6: Incremental Mass Curve of Runoff

(a)	(b)	(c)	(d)	(e)
Time (hours):	Rainfall Distribution:	Total Rainfall (inches):	Accumulated Mass Curve of Runoff* (inches):	Incremental Mass Curve of Runoff

(inches):

0	0	0.00		
1	*0.009*	0.33		0.0000
2	*0.018*	0.67	0.0104	0.0104
3	*0.026*	1.00	0.0833	0.0729
4	*0.035*	1.33	0.2083	0.1250
5	*0.044*	1.67	0.3712	0.1629
6	0.053	2.00	0.5625	0.1913
7	*0.180*	6.83	4.5409	3.9784
8	*0.307*	11.67	9.1240	4.5831
9	*0.434*	16.50	13.8378	4.7139
10	*0.561*	21.33	18.6012	4.7634
11	*0.689*	26.17	23.3886	4.7874
12	0.816	31.00	28.1894	4.8008
13	*0.838*	31.83	29.0181	0.8287
14	*0.860*	32.67	29.8470	0.8289
15	*0.882*	33.50	30.6761	0.8291
16	*0.904*	34.33	31.5054	0.8293
17	*0.925*	35.17	32.3348	0.8295
18	0.947	36.00	33.1645	0.8296
19	*0.956*	36.33	33.4964	0.3319
20	*0.965*	36.67	33.8283	0.3319
21	*0.974*	37.00	34.1603	0.3320
22	*0.982*	37.33	34.4922	0.3320
23	*0.991*	37.67	34.8242	0.3320
24	1.000	38.00	35.1563	0.3320

As a check, use the SCS Runoff Equation (NEH-630.10) determine the total volume of runoff for the $PMP_{24\text{-hour}}$ storm.

$Q = \dfrac{(P - 0.2S)^2}{P + 0.8S}$ where $P = PMP_{24\text{-hour}} = 38.0$ inches; and

$$S = \frac{1000}{CN} - 10 \text{ with } CN = 80 \quad S = \frac{1000}{80} - 10 = 2.5$$

$$Q = \frac{(38.0 - 0.2(2.5))^2}{38.0 + 0.8(2.5)} = \frac{(37.5)^2}{40} = \frac{1406.25}{40} = 35.16 \text{ in}$$ which agrees with the total runoff volume at t = 24.0 hours as shown in column (d) of Table 21B-6.

STEP 4: Develop the freeboard hydrograph.
The development of the freeboard hydrograph follows the procedures in Chapter 16.
a. Determine the time to peak, T_p of the unit hydrograph:
Using equation 16A-10, $T_p = (T_c + \Delta D)/1.7 = (7.1 + 1)/1.7 = 4.76$ hours

b. Determine the qp for a volume of runoff equal to 1-inch.
Using equation 16A-6, $q_p = 484AQ/T_p = 484(15.0)(1)/4.76 = 1525$ cfs

c. Tabulate the ordinates of the unit hydrograph using the ratios for dimensionless unit hydrograph from Table 16-1.
- Columns (a) and (b) in Table 21B-4 show the ratios for computing the dimensionless unit hydrograph from table 16-1.
- Columns (c) and (d) show the computed unit hydrograph.

Table 21B-4: Unit Hydrograph

(a)	*(b)*	*(c)*	*(d)*
Time Ratios: (t/Tp)	*Discharge ratios: (q/Qp)*	*Time (hours): =4.76x (t/Tp)*	*Unit Hydrograph: (cfs): = 1525*(q/Qp)*
0.0	0.000	0	0
0.1	0.030	0.476	45.75
0.2	0.100	0.952	152.5
0.3	0.190	1.428	289.75
0.4	0.310	1.904	472.75
0.5	0.470	2.38	716.75
0.6	0.660	2.856	1006.5
0.7	0.820	3.332	1250.5
0.8	0.093	3.808	141.825
0.9	0.990	4.284	1509.75
1.0	1.000	4.76	1525
1.1	0.990	5.236	1509.75
1.2	0.930	5.712	1418.25
1.3	0.860	6.188	1311.5
1.4	0.780	6.664	1189.5
1.5	0.680	7.14	1037
1.6	0.560	7.616	854
1.7	0.460	8.092	701.5
1.8	0.390	8.568	594.75
1.9	0.330	9.044	503.25
2.0	0.280	9.52	427
2.2	0.207	10.472	315.675
2.4	0.147	11.424	224.175
2.6	0.107	12.376	163.175
2.8	0.077	13.328	117.425
3.0	0.055	14.28	83.875
3.2	0.040	15.232	61
3.4	0.029	16.184	44.225
3.6	0.021	17.136	32.025
3.8	0.015	18.088	22.875
4.0	0.011	19.04	16.775
4.5	0.005	21.42	7.625
5.0	0.000	23.8	0

d. Recompile the unit hydrograph in order that the unit hydrograph and rainfall distribution be tabulated on the same time interval, ΔD. As determined in Step 1 ΔD=1 hour should be used. Table 21B-5 shows the unit hydrograph recompiled on the 1-hour time interval.

Table 21B-5: Unit hydrograph on ΔD =1-hour time increments.

(c) *Time (hours):*	*(d) Unit Hydrograph: (cfs):*
0	0
1	153
2	473
3	1007
4	1419
5	1525
6	1418
7	1189
8	854
9	595
10	427
11	316
12	225
13	164
14	118
15	84
16	61
17	44
18	32
19	23
20	18
21	17
22	16
23	8
24	7

STEP 5: Compute the flood hydrograph.

a. Reverse the incremental runoff table in order to simplify computation of the final hydrograph as shown in Table 21B-7.

Table 21B-7: Reversed Incremental Mass Curve of Runoff.

Time (hours):	Incremental Mass Curve of Runoff (inches):
24	0.3320
23	0.3320
22	0.3320
21	0.3320
20	0.3319
19	0.3319
18	0.8296
17	0.8295
16	0.8293
15	0.8291
14	0.8289
13	0.8287

12	4.8008
11	4.7874
10	4.7634
9	4.7139
8	4.5831
7	3.9784
6	0.1913
5	0.1629
4	0.1250
3	0.0729
2	0.0104
1	0.0000
0	

b. Utilizing the Reversed Incremental Runoff Volume (Table 21B-7) and the Unit Hydrograph (Table 21B-4), compute the flood hydrograph using the method as described in Chapter 16.

Table 21B-8 shows the computation of the final runoff hydrograph which is the design hydrograph for the freeboard hydrograph storm event. See Chapter 16 for a detailed explanation of the computation procedures used to develop the hydrograph.

Table 21B-8. Design Hydrograph - Freeboard Hydrograph (FBH) Storm Event

Freeboard Hydrograph Computation

Time (hours):	Incremental Runoff (Reversed):	Unit Hydrograph	Time (hours)																						
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
0	0.3320	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
1	0.3320	153	0	2	11	19	25	29	608	701	721	728	732	734	127	127	127	127	127	127	51	51	51	51	51	51
2	0.3320	473		0	5	34	59	77	91	1882	2168	2230	2254	2265	2271	392	392	392	392	392	393	157	157	157	157	157
3	0.3320	1007			0	10	73	126	164	193	4006	4614	4746	4796	4820	4834	834	835	835	835	835	835	334	334	334	334
4	0.3319	1419				0	15	103	177	231	271	5643	6501	6687	6757	6791	6810	1175	1176	1176	1176	1177	1177	471	471	471
5	0.3319	1525					0	16	111	191	248	292	6068	6990	7190	7265	7302	7322	1264	1264	1265	1265	1265	1265	506	506
6	0.8296	1418						0	15	103	177	231	271	5643	6501	6686	6757	6791	6810	1175	1176	1176	1176	1177	1177	471
7	0.8295	1189							0	12	87	149	194	228	4732	5452	5607	5666	5694	5710	986	986	986	986	987	987
8	0.8293	854								0	9	62	107	139	163	3398	3914	4026	4068	4089	4100	708	708	708	708	708
9	0.8291	595									0	6	43	74	97	114	2366	2726	2804	2833	2847	2856	493	493	493	493
10	0.8289	427										0	4	31	53	70	82	1701	1959	2015	2036	2046	2052	354	354	354
11	0.8287	316											0	3	23	40	51	60	1258	1449	1490	1506	1513	1518	262	262
12	4.8008	225												0	2	16	28	37	43	893	1029	1059	1070	1075	1078	186
13	4.7874	164													0	2	12	20	27	31	650	749	771	779	783	785
14	4.7634	118														0	1	9	15	19	23	468	539	555	561	563
15	4.7139	84															0	1	6	11	14	16	335	385	396	401
16	4.5831	61																0	1	4	8	10	12	243	280	288
17	3.9784	44																	0	0	3	6	7	8	176	203
18	0.1913	32																		0	0	2	4	5	6	128
19	0.1629	23																			0	0	2	3	4	4
20	0.1250	18																				0	0	1	2	3
21	0.0729	17																					0	0	1	2
22	0.0104	16																						0	0	1
23	0.0000	8																							0	0
24		7																								0
Total Discharge (cfs):			**0**	**2**	**16**	**64**	**172**	**351**	**1166**	**3313**	**7688**	**13957**	**20921**	**27591**	**32737**	**35185**	**34284**	**30888**	**26478**	**22026**	**18082**	**15072**	**12652**	**10570**	**8789**	**7360**

Freeboard Hydrograph Computation

Time (hours):	Incremental Runoff (Reversed):	Unit Hydrograph	Time (hours)																							
			25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
0	0.3320	0																								
1	0.3320	153																								
2	0.3320	473	157																							
3	0.3320	1007	334	334																						
4	0.3319	1419	471	471	471																					
5	0.3319	1525	506	506	506	506																				
6	0.8296	1418	471	471	471	471	471																			
7	0.8295	1189	395	395	395	395	395	395																		
8	0.8293	854	709	283	283	284	284	284	284																	
9	0.8291	595	493	493	197	197	197	197	197	197																
10	0.8289	427	355	355	355	142	142	142	142	142	142															
11	0.8287	316	262	262	262	262	105	105	105	105	105	105														
12	4.8008	225	186	186	186	186	186	75	75	75	75	75	75													
13	4.7874	164	135	136	136	136	136	136	54	54	54	54	54	54												
14	4.7634	118	565	98	98	98	98	98	98	39	39	39	39	39	39											
15	4.7139	84	403	404	70	70	70	70	70	70	28	28	28	28	28	28										
16	4.5831	61	291	293	294	51	51	51	51	51	51	20	20	20	20	20	20									
17	3.9784	44	209	211	212	213	37	37	37	37	37	37	15	15	15	15	15	15								
18	0.1913	32	147	151	153	154	154	27	27	27	27	27	27	11	11	11	11	11	11							
19	0.1629	23	91	105	108	109	110	110	19	19	19	19	19	19	8	8	8	8	8	8						
20	0.1250	18	3	72	83	86	87	87	87	15	15	15	15	15	15	6	6	6	6	6	6					
21	0.0729	17	3	3	68	79	81	82	82	83	14	14	14	14	14	14	6	6	6	6	6	6				
22	0.0104	16	2	3	3	64	74	76	77	78	78	13	13	13	13	13	13	5	5	5	5	5	5			
23	0.0000	8	1	1	1	2	34	39	40	40	40	40	7	7	7	7	7	7	3	3	3	3	3	3		
24		7	0	1	1	1	1	30	34	35	35	36	36	6	6	6	6	6	6	2	2	2	2	2	2	
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Total Discharge (cfs):			**6190**	**5234**	**4354**	**3505**	**2711**	**2038**	**1478**	**1065**	**759**	**522**	**362**	**242**	**176**	**128**	**92**	**63**	**44**	**30**	**22**	**16**	**11**	**5**	**2**	**0**