
ENGINEERING FIELD HANDBOOK (650-EFH)
Chapter 3 (650.03) - Hydraulics

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Acknowledgments

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0300 Introduction

This chapter presents the hydraulic principles that apply to the design and operation of soil and water conservation measures. The chapter contains sections on dimensions and units, principles of water at rest (hydrostatics), and principles of water in motion (hydrokinetics). It also discusses the application of these principles to flow of water in pipes and open channels. The chapter also presents the more common methods of measuring flow of water in open channels and pipes.

0301 Dimensions and Units

The word "dimensions" refers to physical quantities involved in describing a physical system. The basic dimensions of *length* (L), *time* (T), and *mass* (M) can be selected. In the analysis of hydraulic problems many derived quantities are used that combine these basic dimensions. Some of these derived quantities are listed next:

- Velocity: $V = L/T$ (velocity = length/time)
- Acceleration: $a = V/T = L/T^2$ (acceleration = velocity/time)
- Force: $F = M \cdot a = ML/T^2$ (force = mass × acceleration)
- Momentum: $I = M \cdot V = ML/T$ (momentum = mass × velocity)
- Work or energy: $E = F \cdot L = ML^2/T^2$ (work = force × length)
- Power: $P = W/T = ML^2/T^3$ (power = work/time)

The definition of force given above, referred to as *Newton's second law of motion*, indicates that the force F required to provide an acceleration to a body of mass m is given by $F = m \cdot a$. In terms of the basic dimensions (L, T, M), a force has dimensions of ML/T^2, as indicated above. Sometimes, force (F) is used as a basic dimension together with L and T, and mass (M) is considered a derived quantity. In such case, the basic dimensions are (L, T, F) and mass has dimensions of $M = FT^2/L$.

Velocity and acceleration have the same dimensions when either mass or force is considered a basic dimendsion. Using (L, T, and F) as basic dimensions, the following derived quantities are written:

- Momentum: $I = MV = (FT^2/L)(L/T) = FT$
- Work or energy: $E = FL$
- Power: $P = W/T = FL/T$

The basic dimensions and the derived quantities referred to above are described in terms of *units of measurement*. There are two commonly used *systems of units* in modern engineering practice: the *International System* (referred to as S.I., or *Systeme Internationale*, in French) and the *English System* (referred to as E.S., and also known as the Imperial System of units). The International System uses length, time, and mass (L,T,M) as the basic dimensions, while the English System uses length, time, and force (L,T.F) as the basic dimensions. The following table shows the basic units in both systems:

Table 1. Basic units of measurement in the S.I. and E.S. systems

System of units	Length	Time	Mass	Force
International System (S.I.)	meter (*m*)	second (*s*)	kilogram (*kg*)	--
English System (E.S.)	foot (*ft*)	second (*s*)	--	pound (*lb*)

Besides the foot, the *inch* (*in*), the *yard* (*yd*), and the *mile* (*mi*) are commonly-used units of length in the E.S. These units are defined as follows:

$$1 \text{ in} = 1/12 \text{ ft, or } 1 \text{ ft} = 12 \text{ in}$$

$$1 \text{ yd} = 3 \text{ ft}$$

$$1 \text{ mi} = 5280 \text{ ft}$$

To define the unit of force in the S.I. or the unit of mass in the E.S. it is necessary to use the acceleration of gravity (*g*), a quantity that is essentially constant on the surface of the Earth and has the value

$$g = 32.2 \text{ ft/s}^2 = 9.806 \text{ m/s}^2$$

With this acceleration we can define the *weight* of a given mass M as:

- Weight: $W = M \cdot g$ (weight = mass × gravity)

This relationship allows the definition of a unit of force in the S.I., the *newton (N)*, defined as:

$$1 \text{ N} = (1 \text{ kg})\,(1 \text{ m/s}^2) = 1 \text{ kg}\cdot\text{m/s}^2$$

Similarly, by using the expression for mass in terms of weight:

- Mass: $M = W/g$

The unit of mass in the E.S., the *slug*, can be defined as:

$$1 \text{ slug} = (1 \text{ lb})/\,(1 \text{ ft/s}^2) = 1 \text{ lb}\cdot\text{s}^2/\text{ft}$$

In the United States, the English System is the most commonly used system of units. Therefore, most of the problems presented in this chapter are worked using units of the English System. The following are basic units of the English System for the derived quantities presented earlier:

- Velocity: 1 ft/s = 1 fps
- Acceleration: 1 ft/s^2
- Mass: $1 \text{ slug} = 1 \text{ lb}\cdot\text{s}^2/\text{ft}$.
- Momentum: $1 \text{ slug}\cdot\text{ft/s} = 1 \text{ slug}\cdot\text{fps} = (1 \text{ lb}\cdot\text{s}^2/\text{ft})(1 \text{ ft/s}) = 1 \text{ lb}\cdot\text{s}$
- Work or energy: 1 lb·ft
- Power: 1 lb·ft/s

The basic units of the International System for the derived quantities are as follows:

- Velocity: 1 m/s
- Acceleration: 1 m/s^2
- Force: 1 N
- Momentum: 1 N·m/s
- Work or energy: 1 J = 1 $N \cdot m/s^2$ (joule)
- Power: 1 W = 1 J/s (watt)

The International System of units uses also a number of prefixes to indicate decimal fractions or multiples of a given unit. Some of those prefixes are listed below:

- Kilo (k) 10^3 = 1 thousand
- Deci (d) 10^{-1} = 0.1 = one tenth
- Centi (c) 10^{-2} = 0.01 = one hundredth
- Milli (m) 10^{-3} = 0.001 = one thousandth

For example, a commonly used unit for measuring travel distance is the *kilometer* (1 km = 10^3 m = 1000 m), while small pipe diameters could be measured in *centimeters* (1 cm = 10^{-2} m = 0.01 m).

The units of area and volume (e.g., for the measurement of flows) are also of interest in hydraulic applications. The basic units of area and volume in the E.S. are the square foot (1 ft^2) and the cubic foot (1 ft^3); however, other units of area and volume are also used:

$$1 \text{ acre (Ac)} = 43560 \text{ ft}^2$$

$$1 \text{ acre-ft (Ac-ft)} = 43560 \text{ ft}^3$$

$$1 \text{ ft}^3 = 7.48 \text{ gallons (gal)}$$

A derived quantity commonly used in hydraulics is the *discharge* or *flow rate*, Q, defined as vol/T and can be calculated by multiplying velocity, V, times flow area, A, as:

- Discharge or flow rate, $Q = vol/T = V \cdot A$

The basic unit of discharge in the E.S. is 1 ft^3/s commonly referred to as *1 cfs* (cubic feet per second). As an alternative, the discharge in large rivers is sometimes measured in *acre-ft/day*.

While the basic units of area and volume in the S.I. are the square meter and cubic meter, other units are also used:

$$1 \text{ hectare (ha)} = 10000 \text{ m}^2$$

$$1 \text{ m}^3 = 1000 \text{ liters } (l)$$

Exhibit 1 provides a list of basic dimensions and units for the English (E.S.) and International (S.I.) Systems of units.

0302 Unit Conversions

Unit conversions are straightforward if the *conversion factors* are known. Some conversion factors were provided in the previous section. For example, given that 1 ft^3 = 7.48 gal, 1 min = 60 s, and that a discharge is reported to be 0.5 cfs (0.5 ft^3/s), determine the value of the discharge in gallons per minute (gpm). This unit conversion can be accomplished as follows:

$$0.5 \frac{\cancel{ft^3}}{\cancel{s}} \times \frac{7.48\, gal}{\cancel{ft^3}} \times \frac{60\, \cancel{s}}{\text{min}} =$$

$$= 0.5 \times 7.48 \times 60\, gpm = 224.4\, gpm$$

Exhibit 2 provides a list of commonly-used conversion factors for the English (E.S.) and International (S.I.) System of units. A number of unit conversion programs and spreadsheets are available for quick unit conversions.

0303 Dimensional Homogeneity in Equations

In the analysis of hydraulic systems it is necessary to use a number of equations. Most equations are *dimensionally homogeneous*, meaning that the dimensions of both sides of the equation are the same. Equations that define a derived quantity (for example, Newton's second law of motion, F = M·a), are, by definition, dimensionally homogeneous. For other equations, replacing the dimensions of the different variables using either length, time, and mass (L, T, M) or length, time, and force (L, T, F), will verify the dimensional homogeneity of the expression.

The following example verifies the dimensional homogeneity of the equation used to define power in pipe flow (see section *0330 – Pipe Flow*). The power P required to transport a discharge Q of water through a pipeline, with an energy head loss H, is given by:

$$P = \omega Q H \qquad \textbf{[Eq. 1]}$$

where ω is the *specific weight* of water. Exhibit 3 presents the letters of the Greek alphabet, indicating those most commonly used in hydraulic equations. The specific weight of any material is defined as the *weight per unit volume* of the material:

$$\omega = \frac{W}{Vol} \qquad \textbf{[Eq. 2]}$$

where *W* represents weight (in the English system, the pound, *lb*, is commonly used to measure weight), and *Vol* represents volume. In dimensional terms, using (L, T, and F) as basic dimensions, one can write:

$$[\omega] = \frac{[W]}{[Vol]} = \frac{F}{L^3} = FL^{-3}$$

while $[Q] = L^3/T = L^3T^{-1}$, and $[H] = L$. Thus, from the equation defining power, one can write:

$$[P] = [\omega]\,[Q]\,[H] = (FL^{-3})\,(L^3T^{-1})\,(L) = FLT^{-1}$$

Exhibit 1 shows that the dimensions of $[P] = FLT^{-1}$; thus, the equation $P = \omega QH$ is shown to be dimensionally homogeneous.

<u>Manning's equation</u> (see section 0321.2) illustrates an equation that is not dimensionally homogeneous. The equation is given by:

$$V = \frac{C_u}{n} R^{2/3} S_0^{1/2} \qquad \textbf{[Eq. 3]}$$

where *V* is the velocity, C_u is a constant, *R* is the hydraulic radius and has dimensions of length, and S_o is the channel bed longitudinal slope and is a *dimensionless* quantity, and *n* is the Manning's resistance coefficient, another dimensionless quantity.

From the equation, one may obtain the dimensions of V as:

$$[V] = L^{2/3}$$

However, from Exhibit 1, $[V] = LT^{-1}$. Thus, Manning's equation is not dimensionally homogeneous. Although not dimensionally homogeneous, the empirical Manning's equation has proven to be very successful in modeling flow in open channels and pipelines.

0304 Physical Properties of Water

Exhibit 4 presents tables with physical water properties (values may be interpolated for different temperatures) defined as follows:

Density is the mass per unit volume:

$$\rho = \frac{M}{Vol} \qquad \textbf{[Eq. 4]}$$

Units of density in the S.I. are kg/m^3, while in the E.S., they are $slug/ft^3$.

Specific Weight is weight per unit volume (see equation 2):

$$\omega = \frac{W}{Vol}$$

Units of specific weight in the S.I. are N/m^3, while in the E.S., they are lb/ft^3 or *pcf* (pounds per cubic feet).

Because weight is defined as W = Mg, where g is the acceleration of gravity, one can also write $\omega = (W/Vol) = (M{\cdot}g/Vol) = (M/Vol){\cdot}g = \rho{\cdot}g$:

$$\omega = \rho \cdot g \qquad \textbf{[Eq. 5]}$$

Specific gravity is the ratio of the density of any fluid to that of water at 39.2°F (4°C). Water density tends to increase with decreasing temperature to a maximum value at 39.2°F (4°C). As the temperature decreases further, the density of water decreases until the water turns into ice at 32°F or 0°C. Referring to the density of water at 39.2°F or 4°C as $\rho_w = 1000\ kg/m^3 = 1.94\ slug/ft^3$, or to its corresponding specific weight as $\omega_w = 9806\ N/m^3 = 62.4\ lb/ft^3$, the specific gravity of any fluid of density ρ or specific weight ω, is defined as:

$$S = \frac{\rho}{\rho_w} = \frac{\omega}{\omega_w} \qquad \textbf{[Eq. 6]}$$

For example, mercury (chemical symbol = Hg), a liquid metal often used in pressure measurements in hydraulic laboratories, has a specific weight, $\omega = 846.14\ lb/ft^3$. Therefore, the specific gravity of mercury is

$$S = \frac{\omega}{\omega_w} = \frac{846.16\, lb/\ ft^3}{62.4\, lb/\ ft^3} = 13.6$$

Viscosity is a property that measures the ability of water to resist shear deformation. The definition of viscosity can be understood by referring to an experiment that was first conducted by Newton, in which a moving wall is driven at a constant velocity, V, through a quiescent water tank. A schematic top view is shown below:

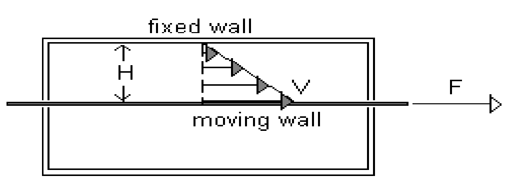


Figure 1. Schematic of Newton's viscosity experiment

Measurements indicate that the local velocity varies linearly from zero at the fixed wall to V at the moving wall. Let F represent the force required to pull the moving wall, and A be the area of the wall in contact with the fluid. The *shear stress* on the moving wall is defined as:

$$\tau = \frac{F}{A} \qquad \textbf{[Eq. 7]}$$

Newton discovered that the shear stress is related to the velocity V and to the length H in the tank through the following relationship:

$$\tau = \mu \frac{V}{H} \qquad \textbf{[Eq. 8]}$$

In this equation, the quantity μ is referred to as the *absolute* or *dynamic viscosity* of water. The *kinematic viscosity* is defined by:

$$\nu = \frac{\mu}{\rho} \qquad \textbf{[Eq. 9]}$$

The units of absolute or dynamic viscosity are $N{\cdot}s/m^2 = kg/(m{\cdot}s)$ in the S.I., and $lb{\cdot}s/ft^2$ in the E.S. And, kinematic viscosity units are m^2/s in the S.I. and ft^2/s in the E.S. Notice that the units of kinematic viscosity are those of area per unit time. The dimensions of kinematic viscosity are $[\nu] = L^2T^{-1}$, i.e., they involve only *kinematic* quantities (area, which is length squared, and time), hence the name *kinematic viscosity*.

For example, at a temperature of 70^{o}F, the kinematic viscosity of water is $\nu = 1.059\text{x}10^{-5}$ ft^2/s, and its specific weight is $\omega = 62.3$ lb/ft^3. To determine the absolute or dynamic viscosity of water at that temperature, we use the formulas $\rho = \omega/g$, and $\mu = \rho\nu = (\omega/g)\nu = \omega\nu/g$, where g = 32.2 ft/s^2:

$$\mu = \frac{\omega\nu}{g} = \frac{\left(62.3\frac{lb}{ft^3}\right)\left(1.059\times10^{-5}\ \frac{ft^2}{s}\right)}{\left(32.2\,\frac{ft}{s^2}\right)} = 2.05\times10^{-5}\ \frac{lb\times\ ft^2\ \times s^2}{ft^4\times s} = 2.05\times10^{-5}\ \frac{lb\cdot s}{ft^2}$$

In the technical literature there are references to two old units of viscosity, the *poise* (*P*), a unit of dynamic viscosity, and the *stokes* (*St*), a unit of kinematic viscosity. The conversion factors for these units into units of the E.S. are:

$$1\text{ P} = 2.0885\times10^{-3}\text{ lb}\cdot\text{s/ft}^2$$

$$1\text{ St} = 1.076\times10^{-3}\text{ ft}^2\text{/s}$$

In pipe flow, viscosity is used to define a quantity known as the *Reynolds number*:

$$R_e = \frac{\rho VD}{\mu} = \frac{VD}{\nu} \qquad \textbf{[Eq. 10]}$$

For example, to determine the Reynolds number in pipe flow in which water at 80°F is flowing at a velocity V = 0.3 ft/s through a pipeline of 6-inch diameter (D = 6 in = 6/12 ft = 0.5 ft), the value of the kinematic viscosity from Exhibit 4 is $\nu = 9.30\times10^{-6}$ ft^2/s. The Reynolds number is calculated as:

$$R_e = \frac{VD}{\nu} = \frac{\left(0.3\frac{ft}{s}\right)(0.5\ ft)}{\left(9.30\times10^{-6}\ \frac{ft^2}{s}\right)} = 16129.03$$

The Reynolds number is a dimensionless number, i.e., a number without units or dimensions.

Vapor pressure and cavitation. Vapor pressure is the ambient (air) pressure at which water boils. For example, at mean sea level, where the atmospheric pressure is 14.697 psi (pounds per square inch), water boils at 212°F. However, at an elevation of 5000 ft (1524 m), where the atmospheric pressure is 12.23 psi, water boils at a temperature of approximately 203°F. Thus, at 212°F, the vapor pressure of water is 14.697 psi, whereas at 203°F, the vapor pressure of water is 12.23 psi. Exhibit 4 provides a table of vapor pressures for water at different temperatures. Exhibit 5 provides a table of the atmospheric pressure variation with elevation.

In some pipe flows or within valves and other appurtenances, the local pressure may fall below that of the vapor pressure of water at a given temperature. In such locations it is possible to develop small vapor cavities that, when swept by the flow towards locations of higher pressure, may collapse onto themselves generating such force in the process that it may chip away at the pipe or valve walls. This condition is known as *cavitation* (see section 0335); it should be avoided to prevent damage to pipes or appurtenances. Figure 2, below, shows cavitation damage on the propeller of a Francis turbine.

Figure 2. Cavitation damage on the propeller of a Francis turbine.

Modulus of elasticity and water hammer. *Bulk modulus of elasticity* is a measure of resistance of water to volume change under effect of pressure. Water and liquids, in general, are referred to as *incompressible fluids* because they experience very small volume changes when subject to very large pressure changes. Gases, on the other hand, have large volume changes when pressure changes, and are, therefore, referred to as *compressible fluids*. For any material, the bulk modulus of elasticity is defined as:

$$E = -\frac{\Delta p}{\Delta V / V} \qquad \textbf{[Eq. 11]}$$

where Δp is the change in pressure applied to the material with initial volume V, and ΔV is the resulting change in volume. The negative sign in the equation signifies that an increase in pressure (positive Δp) will produce a decrease in volume (negative ΔV). Thus, the bulk modulus of elasticity can be defined as the change in pressure per unit change in volume of a fluid.

For example, at a temperature of 60^{o}F, the modulus of elasticity of water is E = 311000 psi = 44784000 psf. If we wanted to reduce the volume of a water mass by $\Delta V = -0.1$ ft^3, and given that the original volume is $V = 1$ ft^3, the equation above indicates that the required pressure increase is extremely large, namely:

$$\Delta p = -E \cdot \frac{\Delta V}{V} = -(311000\ psi) \cdot \frac{(-0.1\ ft^3)}{(1\ ft^3)} = 31100\ psi$$

This calculation illustrates that water (and liquids, in general) are, for most practical purposes, incompressible. The incompressibility of water is a factor in producing a phenomenon known as *water hammer* (see section 0334).

0310 Hydrostatics

Hydrostatics refers to the study of water (and other liquids) at rest. Such is the case, for example, of water contained in a storage tank with no flow in or out. Subjects of interest in hydrostatics include determination of pressures in a fluid, instruments for measuring pressure, calculation of forces on submerged surfaces (e.g., on gates or tank walls), and study of buoyancy.

0311 Hydrostatic Pressure Relationships

Pressure is defined as the force per unit area that a fluid (liquid or gas) exerts on a surface submerged in the fluid. The pressures discussed here are those within a body of water at rest. Consider, for example, a water tank as shown in the figure below.

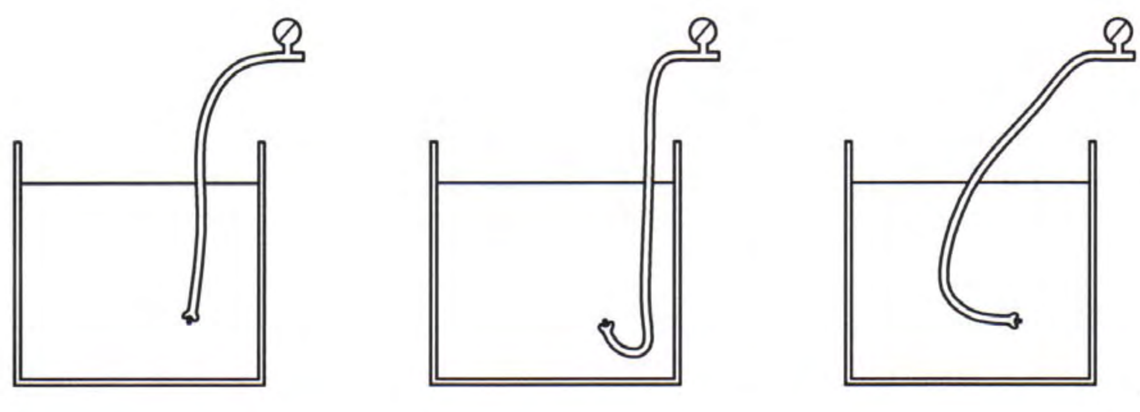


Figure 3. Schematic of pressure measurement in a liquid.

A small probe of area *A* is introduced at a point in the tank as shown. If the force that the water exerts on the probe tip is *F*, then the pressure at that point is

$$p = \frac{F}{A} \quad \textbf{[Eq. 12]}$$

Changing the orientation of the probe's tip at the same point, does not change the force exerted by the water on the tip. Thus, the pressure at any given point does not change with the orientation of the surface acted upon. Pressure is said to be *isotropic*, i.e., it is independent of the orientation in which it is measured.

The following table lists some of the most commonly used units of pressure in the English System (E.S.) and International System (S.I.). Notice that pressure can also be expressed in terms of the height of a liquid column as shown below. In this table, H_20 and Hg are the chemical symbols of water and mercury, respectively.

Unit of Pressure	Definition or Equivalent
Basic Units	
psf	1 lb/ft^2 = 0.006944 psi = 47.88 Pa
psi	1 lb/in^2 = 144 psf = 6894.76 Pa
Pa (Pascal)	1 N/m^2 = 0.02088 psf = 0.000145 psi
Other Units	
kPa (kilo Pascal)	1,000 Pa
MPa (mega Pascal)	1,000 kPa = 1,000,000 Pa
bar	14.50 psi = 100,000 Pa
mb (millibar)	0.001 bar = 0.014504 psi = 100 Pa
atm (atmosphere)	14.7 psi = 101,325 Pa
Height of Liquid Column	
1 m H_20	1.4209 psi = 9796.85 Pa
1 ft H_20	0.4331 psi = 2986.08 Pa
1 in H_20	0.03609 psi = 248.84 Pa
1 mm Hg	0.01934 psi = 133.32 Pa
1 in Hg	0.4912 psi = 3,386.39 Pa

Pressure, within a liquid at rest, increases linearly with depth. Referring to Figure 4, if the pressure at elevation z_0 is known to be p_0, then the pressure p at elevation z, is given by the hydrostatic law:

$$p = p_0 + \omega \Delta z$$

where ω is the specific weight of the liquid, and $\Delta z = z - z_0$ is the difference in elevation of the two points of interest. The elevations z and z_0 are measured from any common horizontal level or datum.

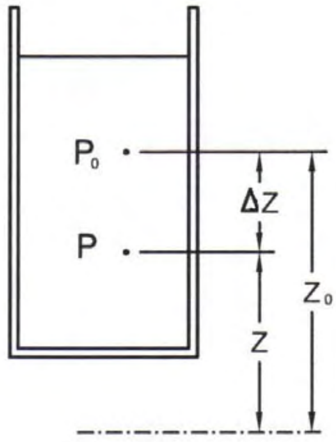


Figure 4. Pressures within a liquid at rest.

Atmospheric pressure refers to the pressure exerted by the weight of the air in the atmosphere. As there is greater weight of air above lower than higher levels on the surface of the earth, atmospheric pressure decreases as elevation increases. Exhibit 5 shows the typical values of atmospheric pressure at different elevations above mean sea level. Atmospheric pressure is measured with an instrument called a *barometer*. Typical values at mean sea level and at a temperature of 59°F (15°C) are

$$p_{atm} = 1\ atm = 2116.22\ psf = 14.70\ psi = 101.33\ kPa$$

$$p_{atm} = 760\ mmHg = 407.19\ in\ H_2O = 29.92\ in\ Hg$$

Absolute pressure and gage pressure. Absolute pressure refers to the pressure measured with a zero value corresponding to a perfect vacuum, i.e., the total absence of matter in a volume. Absolute pressure, therefore, is always a positive quantity. Barometric pressure is an example of absolute pressure. To emphasize that a certain quantity is reported in units of absolute pressure, sometimes "*a*" or "*abs*" is added to the units of pressure. For example, atmospheric pressure is written as:

$$p_{atm} = 2116.22\ psfa = 14.70\ psia = 101.33\ \text{kPa-}abs$$

$$p_{atm} = 760\ mmHg\text{-}abs = 407.19\ in\ H_2O\text{-}abs = 29.92\ in\ Hg\text{-}abs$$

On the other hand, when measuring pressure with *manometers* (see next section), it is possible to shift the zero value of the scale to the level of atmospheric pressure. Thus, in this *gage pressure* scale, pressures above atmospheric will be positive, while those below atmospheric will be negative.

Absolute and gage pressures are related by the following equation:

$$p_{abs} = p_{gage} + p_{atm} \qquad \textbf{[Eq. 13]}$$

For example, if the atmospheric pressure is $p_{atm} = 13.5\ psi$ at a given location, and if a pressure gage on a pipe reads $p_{gage} = -12\ psi$, then the corresponding absolute pressure will be:

$$p_{abs} = p_{gage} + p_{atm} = -12\ psi + 13.5\ psi = 1.5\ psi$$

Gage pressure distribution in liquids. Consider a tank open to the atmosphere. The gage pressure at the free surface of the tank would be zero, by definition. The gage pressure at any point located at a depth h below the free surface will be given by

$$p_{gage} = 0 + \omega \cdot h = \omega \cdot h \qquad \textbf{[Eq. 14]}$$

If the tank is closed and the free surface is pressurized at pressure p_s, then the pressure at a point at depth h below the free surface will be subject to a pressure given by

$$p = p_s + \omega \cdot h$$

The pressure p, calculated above, would be an absolute or a gage pressure depending on whether p_s is given as an absolute or gage pressure, respectively.

Example 1 – Pressure at depth in water

As an example, consider the water contained in a reservoir to a depth of 20-ft, as illustrated in Figure 5 below.

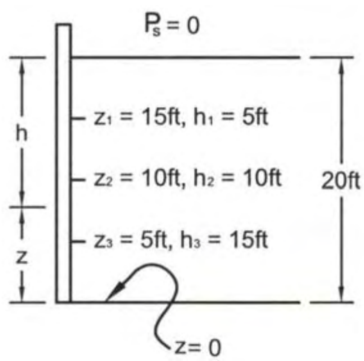


Figure 5. Calculation of pressures at different depths.

Calculating the pressure at points 1, 2, and 3, located at elevations $z_1 = 15\ ft$, $z_2 = 10\ ft$, and $z_3 = 5\ ft$, measured from the reservoir's bottom, requires that the water depths be calculated first:

$$h_1 = 20\ ft - z_1 = 20\ ft - 15\ ft = 5\ ft$$

$$h_2 = 20\ ft - z_2 = 20\ ft - 10\ ft = 10\ ft$$

$$h_3 = 20\ ft - z_3 = 20\ ft - 5\ ft = 15\ ft$$

Then, using ω = 62.4 *lb/ft³* for the specific weight of water, the corresponding (gage) pressures are:

$$p_1 = \omega \cdot h_1 = 62.4 \frac{lb}{ft^3} \times 5\ ft = 312 \frac{lb}{ft^2} = \frac{312}{144} \frac{lb}{in^2} = 2.2\ psi$$

$$p_2 = \omega \cdot h_2 = 62.4 \frac{lb}{ft^3} \times 10\ f\ = 624 \frac{lb}{ft^2} = \frac{624}{144} \frac{lb}{in^2} = 4.3\ psi$$

$$p_3 = \omega \cdot h_3 = 62.4 \frac{lb}{ft^3} \times 15\ f\ = 936 \frac{lb}{ft^2} = \frac{936}{144} \frac{lb}{in^2} = 6.5\ psi$$

0311.1 Piezometers and Manometers

Manometers are instruments used in the measurement of pressure. The simplest manometer consists of a u-tube with a leg attached to the point where the pressure will be measured, and the other leg open to the atmosphere. Such a manometer is illustrated in Figure 6.

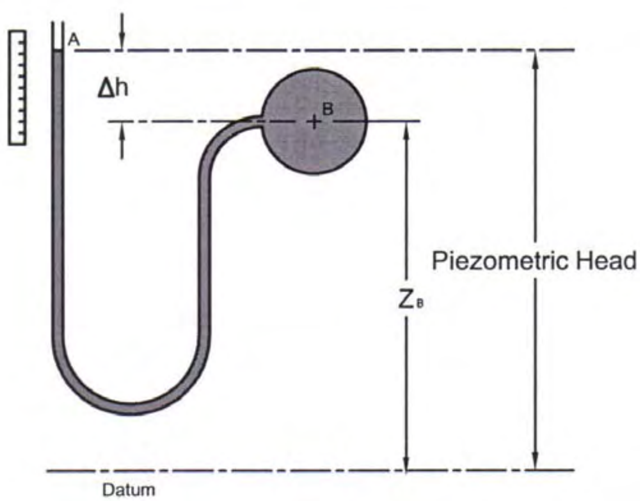


Figure 6. Simple manometer (piezometer)

The circle centered at B may represent, for example, the cross-section of a flowing pipe. The elevation of point A (open to the atmosphere) with respect of the pipe centerline B is equal to *Δh*. Suppose that the liquid (e.g., water) in the pipe and manometer has a specific weight ω, then, according to the hydrostatic law, the pressure at B is given by:

$$p_B = p_A + \omega \cdot \Delta h$$

If we use gage pressure to report our result, then we can take $p_A = 0$, and the pressure at the pipe centerline will be simply:

$$p_B = \omega \cdot \Delta h$$

In pipeline flow we are often interested in determining the so-called *piezometric head* of the flow at a given location. The piezometric head, as illustrated in Figure 6, above, is the sum of the *pressure head* ($\Delta h = p_B/\omega$) plus the elevation of the pipe centerline z_B. Thus, a manometer, as the one illustrated above, is also known as a *piezometer*, for it shows the piezometric head in a pipe flow.

In many cases, piezometers are simply vertical tubes attached to the top of the pipeline, as illustrated in the figure below.

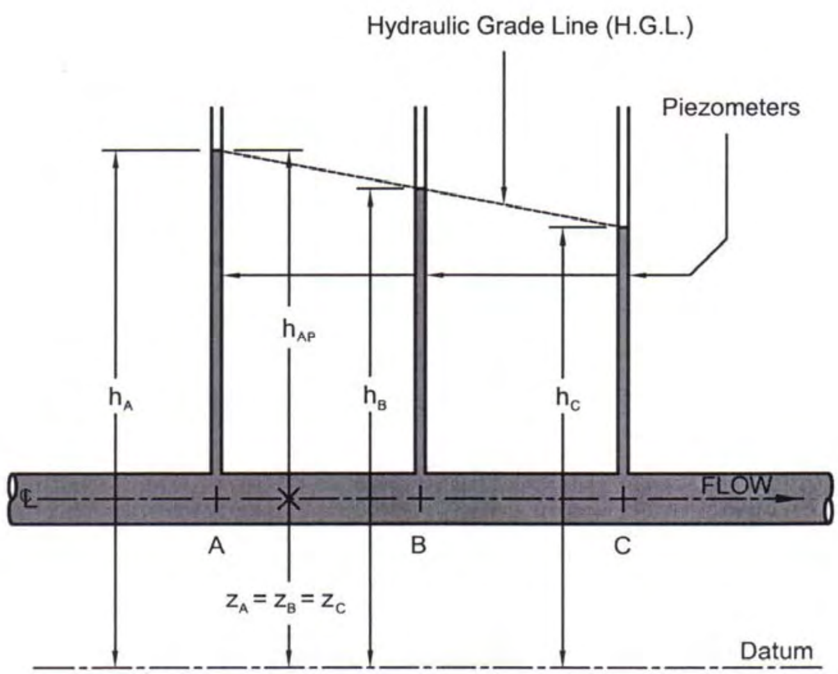


Figure 7. Piezometers on a horizontal pipe flow.

The piezometers in Figure 7 above show the location of the *hydraulic grade line (HGL)*, which, in a horizontal pipeline, illustrates the decrease in pressure along the pipeline in the direction of the flow. The piezometers in the figure show that the piezometric head decreases from A to B to C, thus, $h_A > h_B > h_C$. The centerline elevation of points A, B, and C is the same, i.e., $z_A = z_B = z_C$, therefore, the pressure heads (for point A, the pressure head is $h_A - z_A$) will be such that $h_{AP} > h_{BP} > h_{CP}$.

The photograph in Figure 8, below, shows piezometers located before and after an orifice meter in a transparent pipeline, typically used in a laboratory setting. An orifice meter is used to measure flow discharge in a pipe. The piezometers are used to measure the pressure variation about the orifice meter.

Figure 8. Piezometers near an orifice meter in a pipeline

The figure below shows a u-tube manometer being used to determine the pressure at the centerline of a flowing pipe (point C). The flowing fluid has a specific weight ω_1, while the *manometric fluid* has a specific weight ω_2.

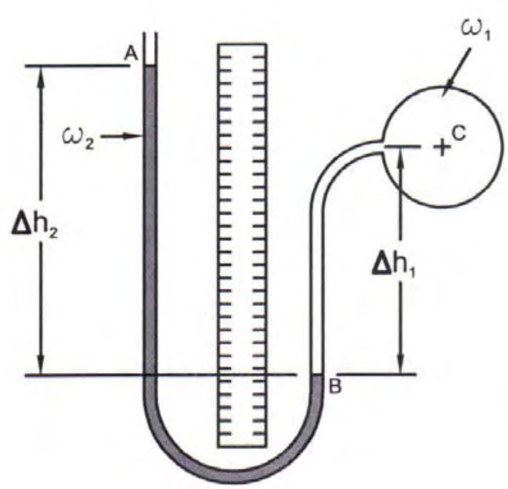


Figure 9. U-tube manometer

For the case shown in Figure 9, above, point A is open to the atmosphere, thus, using gage pressures, we can write $p_A = 0$. The interface between the two liquids, point B, is called a *meniscus*. Point B is located at a depth Δh_2 with respect to the free surface meniscus A. Thus, the gage pressure at point B can be calculated as:

$$p_B = p_A + \omega_2 \cdot \Delta h_2 = 0 + \omega_2 \cdot \Delta h_2 = \omega_2 \cdot \Delta h_2$$

On the other hand, within the flowing fluid, the pressure at point B can be written as (hydrostatic law):

$$p_B = p_C + \omega_1 \cdot \Delta h_1$$

Equating the two expressions found above for p_B one can write:

$$p_C + \omega_1 \cdot \Delta h_1 = \omega_2 \cdot \Delta h_2$$

from which it follows that:

$$p_C = \omega_2 \cdot \Delta h_2 - \omega_1 \cdot \Delta h_1$$

If the flowing fluid is water, $\omega_1 = \omega_w$ (the specific weight of water), and $\omega_2 = S_m \omega_w$, where S_m is the specific gravity of the manometric fluid (e.g., for mercury, $S_m = 13.56$). Thus, one can write:

$$p_C = S_m\ \omega_w \cdot \Delta h_2 - \omega_w \cdot \Delta h_1 = \omega_w\ (S_m\ \Delta h_2 - \Delta h_1)$$

For example, to determine the pressure at point C in the figure above, given that the manometric fluid is mercury ($S_m = 13.56$), and that Δh_2 = 6 *in* = 6/12 *ft* = 0.5 *ft*, Δh_1 = 2 *in* = 2/12 *ft* = 0.167 *ft*, and ω_w = 62.4 lb/ft^3, using the equation above:

$$p_C = \omega_w\ (S_m\ \Delta h_2 - \Delta h_1) = (62.4\ lb/ft^3)(13.56\times 0.5\ ft - 0.167\ ft) = 412.7\ lb/ft^2, \text{ or}$$

$$p_C = \frac{412.7}{144}\frac{lb}{in^2} = 2.87\ psi \approx 3\, psi$$

The photograph in Figure 10 shows a u-tube manometer used to measure flow discharge in the pipe located behind the manometer. The manometer legs in the photograph are attached to tubes connected to a Venturi meter (not shown). The manometric fluid shown is a red manometric fluid with a specific gravity S_m = 0.75.

Figure 10. U-tube manometer for flow measurement in a pipeline.

Rules for manometer calculations. The following rules apply to the calculation of pressures in tube manometers involving a number of fluids:

- Start at a point of known pressure or at a point where the pressure is required.
- Following the tube manometer to the next meniscus, add the product *(specific weight× depth)* if the meniscus is located below the starting point, or subtract the same product if the meniscus is located above the starting point.
- Continuing the path to the next meniscus, add the product (*specific weight × depth*) if the meniscus is located below the previous meniscus, or subtract the same product if the meniscus is located above the previous meniscus.
- When reaching the ending point in the manometer path, make the resulting sum equal to the pressure at the ending point.

For example, for the piezometer in Figure 6, one can write:

$$p_A + \omega \cdot \Delta h = p_B$$

For the manometer of Figure 9, one can write:

$$p_A + \omega_2 \cdot \Delta h_2 - \omega_1 \cdot \Delta h_1 = p_C$$

As an additional example, consider the case in which a manometer at an orifice plate, illustrated in Figure 11, below.

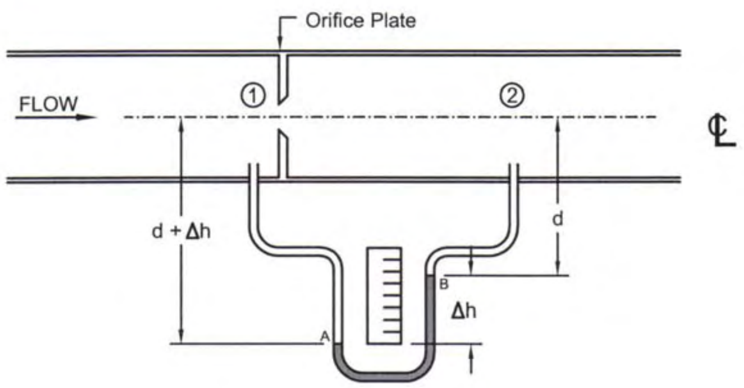


Figure 11. Schematic of manometric measurements at an orifice plate in a pipe.

To determine the difference in pressures between points 1 and 2, use the rules for manometer calculations presented above; start at point 1 and first write:

$$p_1 + \omega_w(d+\Delta h)$$

The above expression reachs from point 1 to meniscus A using water for the specific weight. From point A to point B, use the specific weight of the manometric fluid, and subtract the amount $\omega_m \Delta h$ from the expression above, thus, producing:

$$p_1 + \omega_w(d+\Delta h) - \omega_m \Delta h$$

From point B to point 2, subtract the amount $\omega_w d$ from the expression above, and make the result equal to the pressure at point 2:

$$p_1 + \omega_w(d+\Delta h) - \omega_m \Delta h - \omega_w d = p_2$$

Expanding the expression above:

$$p_1 + \omega_w d + \omega_w \Delta h - \omega_m \Delta h - \omega_w d = p_2$$

Eliminating the terms + $\omega_w d$ and - $\omega_w d$, one gets:

$$p_1 + \omega_w \Delta h - \omega_m \Delta h = p_2$$

Algebraic manipulation of this equation allows writing:

$$\Delta p = p_1 - p_2 = (\omega_m - \omega_w)\, \Delta h$$

The resulting equation can be simplified by introducing the specific gravity of the manometric fluid, i.e., $\omega_m = S_m \omega_w$, thus:

$$\Delta p = \omega_w \, \Delta h \, (S_m - 1)$$

If $\omega_w = 62.4 \; lb/ft^3$, $\Delta h = 8 \; in = 8/12 \; ft = 0.666 \; ft$, and $S_m = 13.56$ (for mercury), the difference in pressure, Δp is:

$$\Delta p = \omega_w \, \Delta h \, (S_m - 1) = (62.4 \; lb/ft^3)(0.666 \; ft)(13.56 - 1) = 521.97 \; lb/ft^2$$

or

$$\Delta p = \frac{521.97}{144} \frac{lb}{in^2} = 3.62 \; psi$$

Orifice plates may be used to measure flow in pipelines. The pressure difference, Δp, can be related to the pipeline discharge by calibration or by theoretical analysis. Examples of such analyses are presented in the next section.

Deformation Manometers. Deformation manometers, such as the Bourdon manometer shown in the following figure, utilize the deformation of spiral tubes or of diaphragms to measure pressure. These manometers are calibrated by manufacturers or in the laboratory and provide a direct reading of the pressure at the manometric tap.

Modern deformation manometers have digital readouts, making the reading of the pressure straightforward. The Bourdon manometer shown in Figure 12 has an analog scale, with the pressure marked by the pointer attached to the spiral tube located inside the manometer.

Figure 12. Deformation manometer (Bourdon manometer)

0312 Forces on Submerged Plane Surfaces

The calculation of the size, direction, and location of the forces on submerged surfaces is essential in the design of dams, bulkheads, water control gates, and other related appurtenances.

Horizontal surface. The hydrostatic law indicates that pressure varies with depth. Thus, a horizontal surface within a liquid at rest is subject to the same pressure p over the entire surface, and the resultant force F on the surface is given by:

$$F = pA = \omega hA$$ **[Eq. 15]**

where, A is the area of the surface.

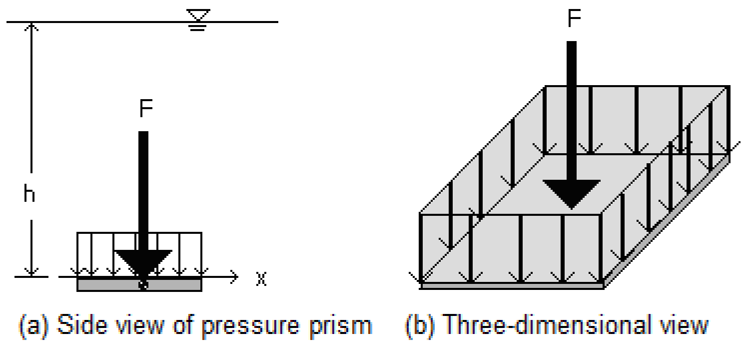


Figure 13. Force on a submerged horizontal surface

Figure 13 shows that the pressure on top of the horizontal surface is represented by vertical arrows of the same height pointing towards the surface. The pressure arrows and the horizontal surface form a three-dimensional figure known as the *pressure prism*. The resultant force vector on any surface coincides with the centroid (also referred to as the center of mass or center of gravity) of the pressure prism. The force on a horizontal surface will be vertical and applied to the centroid C of the surface as illustrated in Figure 14, below. The point of application of the force is referred to as the *center of pressure* P.

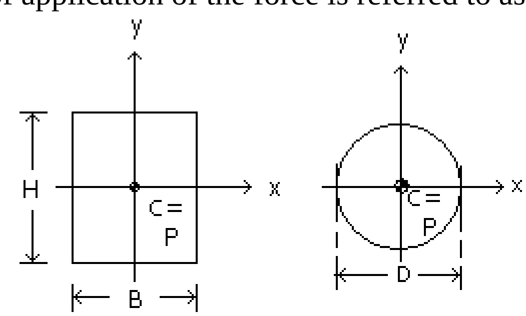


Figure 14. Point of application of a force on a submerged horizontal surface

Example 2 – Hydrostatic force on horizontal area

A circular tank 15 *ft* in diameter is filled with water to a depth of 2 *ft*. Determine the magnitude and location of the vertical force that the water applies on the tank bottom.

The tank has a diameter D = 15 *ft*, therefore, the area of the tank bottom is that of a circle:

$$A = \frac{\pi \cdot D^2}{4} = \frac{3.1416 \times (15\ ft)^2}{4} = 176.71\ ft^2$$

The magnitude of the pressure applied to the bottom of the tank can be calculated by using the specific weight of water, $\omega = 62.4\ lb/ft^3$ and depth of the water in the tank, h = 2 *ft*:

$$p = \omega h = (62.4\ lb/ft^3)(2\ ft) = 124.8\ lb/ft^2$$

The magnitude of the force applied on the tank bottom is:

$$F = pA = (124.8\ lb/ft^2)(176.71\ ft^2) = 22053.41\ lb$$

The force is applied at the center of the circular bottom and is equivalent to the weight of the water.

Inclined surface. For a surface located on an inclined plane, the pressure increases linearly from the top of the surface to the bottom of the surface. The magnitude of the force on the surface is calculated as:

$$F = p_c A = \omega h_c A \quad \textbf{[Eq. 16]}$$

Where $p_c = \omega h_c$ is the pressure at the centroid of the figure. The force, F, is represented by the volume of the pressure prism.

Figure 15, below, shows the pressure distribution along an inclined surface and the resulting force on a rectangular region laid on the inclined surface. The rectangular region could represent a gate on the slope of a dam or dike. The inclined surface is located at an angle θ with respect to the horizontal. Alternatively, the slope can be indicated by the proportion zH(horizontal):1V(vertical) as shown in the figure. For the case illustrated in Figure 15, the angle θ is related to the slope by:

$$\tan(\theta) = \frac{b}{a} = \frac{1}{z} \quad \textbf{[Eq. 17]}$$

Figure 15 shows a system of coordinate axes, x and y, located on the inclined surface. The system is selected so that the x-axis is located along the free surface. Points on the inclined surface can be located by either their depth h or their y coordinate along the surface. These two distances are related by:

$$h = y \cdot \sin(\theta) \quad \textbf{[Eq. 18]}$$

Point 1 is located at the top of the gate while point 2 is located at the bottom of the gate. The gate dimensions are *B* (width) and *H* (height). Point C represents the centroid of the gate, while point P represents the point of application of the hydrostatic force *F* on the gate, the *center of pressure*.

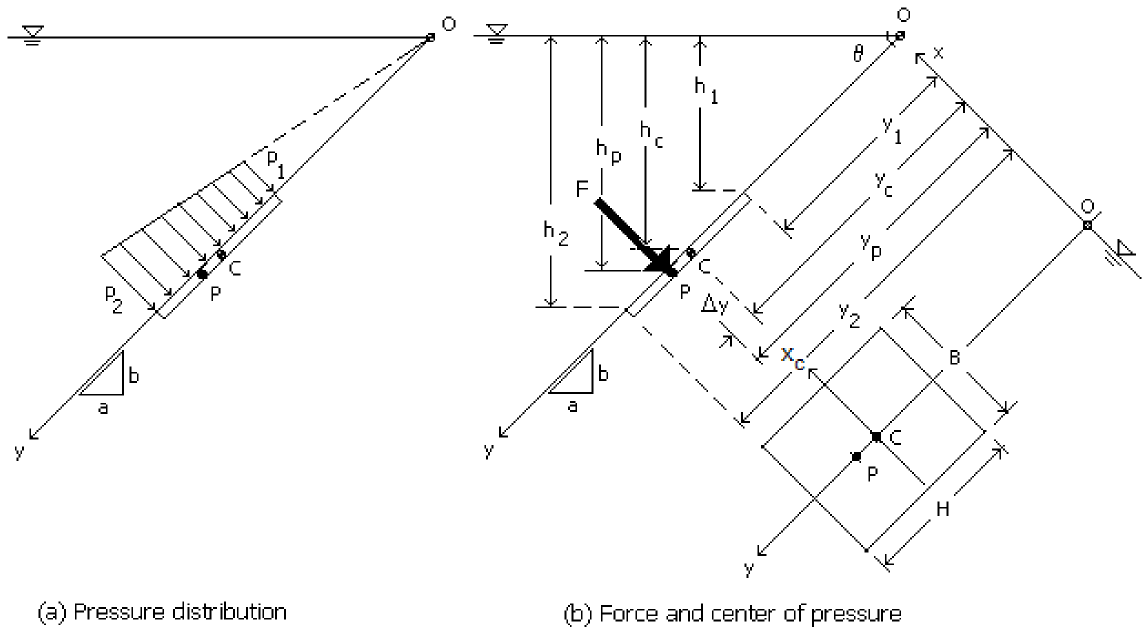


Figure 15. Pressure distribution, force, and center of pressure on an inclined surface.

Unlike the case of a horizontal surface, the center of pressure on an inclined surface is located below the centroid of the surface along the plane of the surface, by a distance given by:

$$\Delta y = \frac{I_c}{A \cdot y_c} \quad \textbf{[Eq. 19]}$$

In this formula, y_c is the location of the centroid of the surface measured from the free surface along the plane of the surface, and I_c is the *moment of inertia* of the surface with respect to a centroidal axis parallel to the *x* axis (i.e., an axis through point C). Thus, the center of pressure will be located at a distance

$$y_p = y_c + \Delta y = y_c + \frac{I_c}{A \cdot y_c} \quad \textbf{[Eq. 20]}$$

For the rectangular and circular figures of Figure 14, with the *x* axis representing the centroidal axis x_c, the centroidal moments of inertia are the following:

Rectangular area: $$I_c = \frac{1}{12} B H^3 \quad \textbf{[Eq. 21]}$$

Circular area: $$I_c = \frac{\pi}{64} D^4$$ **[Eq. 22]**

Since most gates are either rectangular or circular, equations 21 and 22 will be useful for calculating the center of pressure of those types of gates by using equations 19 and 20.

Example 3 – Hydrostatic force on inclined area

For the rectangular gate illustrated in Figure 15, B = 3 *ft*, H = 4.5 *ft*, a = 2, b = 1, y_1 = 5 *ft*, determine the force on the gate and the location of the center of pressure.

The slope is specified by the numbers a = 2 and b = 1, i.e., 2H:1V. The corresponding angle θ is calculated as:

$$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ$$

While the top of the gate is located at a distance y_1 = 5 *ft* measured along the slope, the bottom of the gate will be located at:

$$y_2 = y_1 + H = 5\,ft + 4.5\,ft = 9.5\,ft$$

The centroid is located midway between y_1 and y_2:

$$y_c = \frac{y_1 + y_2}{2} = \frac{5\,ft + 9.5\,ft}{2} = 7.25\,ft$$

The depth of the centroid is:

$$h_c = y_c \sin(\theta) = 7.25\,ft \times \sin(26.57^o) = 3.24\,ft$$

Thus, the pressure at the centroid is (equation 18):

$$p_c = \omega h_c = 62.4\,lb/ft^3 \times 3.24\,ft = 202.18\,lb/ft^2$$

The area of the gate is:

$$A = BH = 3\,ft \times 4.5\,ft = 13.5\,ft^2.$$

The force on the gate (equation 16):

$$F = p_c A = 202.18\,lb/ft^2 \times 13.5\,ft^2 = 2729.43\,lb$$

To calculate the location of the center of pressure, start by calculating the centroidal moment of inertia using equation 21:

$$I_c = \frac{1}{12} BH^3 = \frac{1}{12} \cdot 3\,ft \cdot (4.5\,ft)^3 = 22.78\,ft^4$$

The distance between the centroid C and the center of pressure is calculated with equation 19:

$$\Delta y = \frac{I_c}{A \cdot y_c} = \frac{22.78\,ft^4}{13.5\,ft^2 \cdot 7.25\,ft} = 0.23\,ft\,.$$

The center of pressure is located at the distance (equation 20):

$$y_p = y_c + \Delta y = 7.25\,ft + 0.23\,ft = 7.48\,ft.$$

And the depth of the center of pressure is (equation 18):

$$h_p = y_p \sin(\theta) = 7.48\,ft \times \sin(26.56^o) = 3.35\,ft.$$

Vertical surface. In Figure 16, a rectangular surface of width B is located on a vertical plane and the free surface of the water reaches to a depth H.

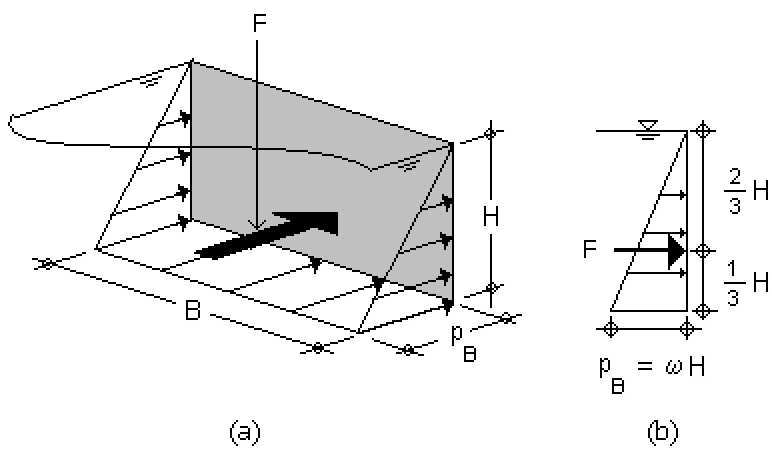


Figure 16. Pressure prism (a) and force location (b) for a vertical rectangular surface

The triangular distribution in Figure 16(b) represents the pressure distribution on the vertical surface. The pressure at the bottom is given by $p_B = \omega H$. The force F on the surface is equal to the volume of the pressure prism:

$$F = \left(\frac{1}{2} \cdot p_B \cdot H\right) \cdot B = \frac{1}{2} \omega BH^2 \qquad \textbf{[Eq. 23]}$$

Using equations 20 and 21 and the area of this rectangular surface, $A = BH$, one can prove that the location of the center of pressure (point of application of the force) is given by:

$$y_p = h_p = \frac{2}{3}H \qquad \textbf{[Eq. 24]}$$

Thus, the force is applied at 2/3 of the depth measured from the surface, or 1/3 of the depth measured from the bottom as indicated in Figure 16 (b). Examples of this type surface include vertical gates and flashboards.

Example 4 – Hydrostatic force on vertical gate

A wooden vertical gate with a width of 10 *ft* is used to close a canal. If the water depth on the gate is 2.5 *ft*, determine the hydrostatic force on the gate and its location.

For this case B = 10 *ft*, H = 2.5 *ft*, and ω = 62.4 lb/ft^3, thus, the force is (equation 23):

$$F = \frac{1}{2}\omega BH^2 = \frac{1}{2}\times 62.4\frac{lb}{ft^3}\times 10\,ft\times(2.5\,ft)^2 = 1950\,lb$$

The force is located at a distance from the surface (equation 24):

$$y_p = h_p = \frac{2}{3}H = \frac{2}{3}\times 2.5\,ft = 1.67\,ft$$

The use of a spreadsheet application greatly facilitates calculation of forces on submerged plane surfaces.

0313 Buoyancy Forces

Buoyancy is the upwards force experienced by solid bodies submerged in liquids or gases. *Archimedes' principle* states that a solid body submerged in a fluid (i.e., liquid or gas) experiences a vertical upward force (the buoyancy force) equal to the weight of the volume of fluid it displaces. Thus, the buoyancy force, F_B, experienced by a body of volume V submerged in a fluid of specific weight ω, is given by:

$$F_B = \omega V \qquad \textbf{[Eq. 25]}$$

0313.1 Buoyancy Applications

A solid body submerged is also acted upon by its own weight, which can be calculated as

$$W = \omega_s V \qquad \textbf{[Eq. 26]}$$

where ω_s is the specific weight (weight per unit volume) of the solid material.

A solid body fully submerged in water is subject to its own weight W (equation 26) and the buoyancy force (equation 25) exerted by the water on the solid body. Figure 16 illustrates several possibilities in terms of force equilibrium when a solid body, fully submerged in water, is released.

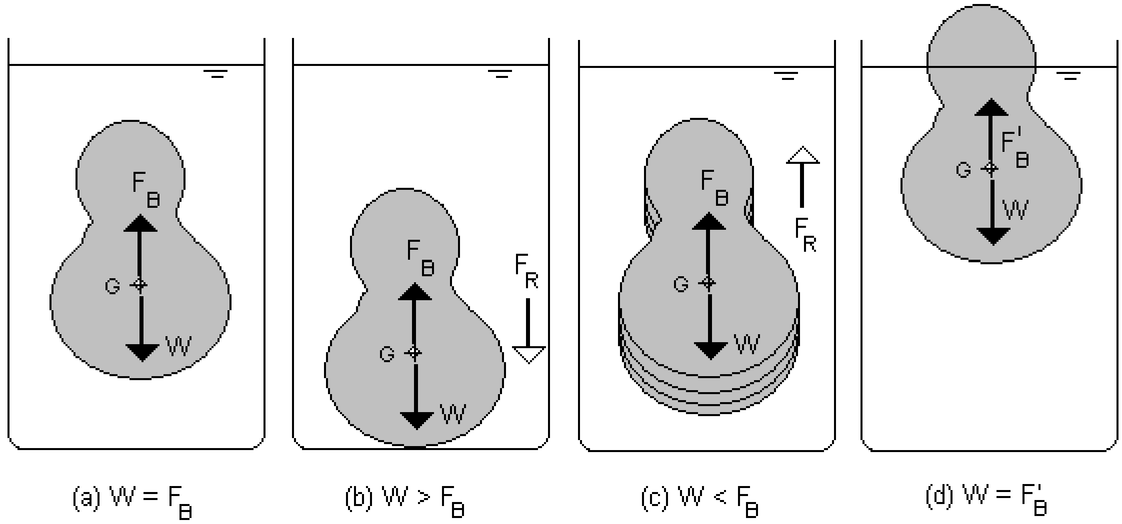


Figure 16. Buoyancy and weight forces acting on a submerged body

If the net resulting force, F_R, on the body, i.e., the difference between its weight W and the buoyancy force F_B, is zero, the body is said to be *neutrally buoyant* and it will remain in place within the water as illustrated in Figure 16(a).

The net resulting force, F_R, is a downward force if the weight is larger than the buoyancy force ($W>F_B$), i.e., if the specific weight of the solid is larger than that of water ($\omega_s > \omega$, e.g., metal). In this case the solid body will sink into the liquid until reaching the bottom of a container, as illustrated in Figure 16(b).

On the other hand, if the weight is smaller than the buoyancy force ($W<F_B$), i.e., if the specific weight of the solid is smaller than that of water ($\omega_s < \omega$, e.g., wood), when released, the body will float upwards towards the free surface as illustrated in Figure 16 (c). A floating body will reach equilibrium when its weight is balanced by an equal buoyancy force F'_B, calculated based on the volume of water displaced below the flotation line (V_b). Therefore, for the case illustrated in Figure 16 (d):

$$W = F'_B = \omega V_b \qquad \textbf{[Eq. 27]}$$

Example 5 – Buoyancy force calculation – depth of a loaded barge

A small barge with a rectangular bottom 4 *ft* by 3 *ft* is to be used to carry 400 *lb* of construction materials. What is the minimum depth needed to carry such weight?

The weight W = 400 *lb* must be balanced by the buoyancy force on the barge, namely, $F'_B = 62.4\ lb/ft^3 \times 4\ ft \times 3\ ft \times h$, thus:

$$h = \frac{400\,lb}{62.4lb/\ ft^3 \times 4\,ft \times 3\,ft} = 0.53\,ft = 0.53 \times 12\ in = 6.36\ in$$

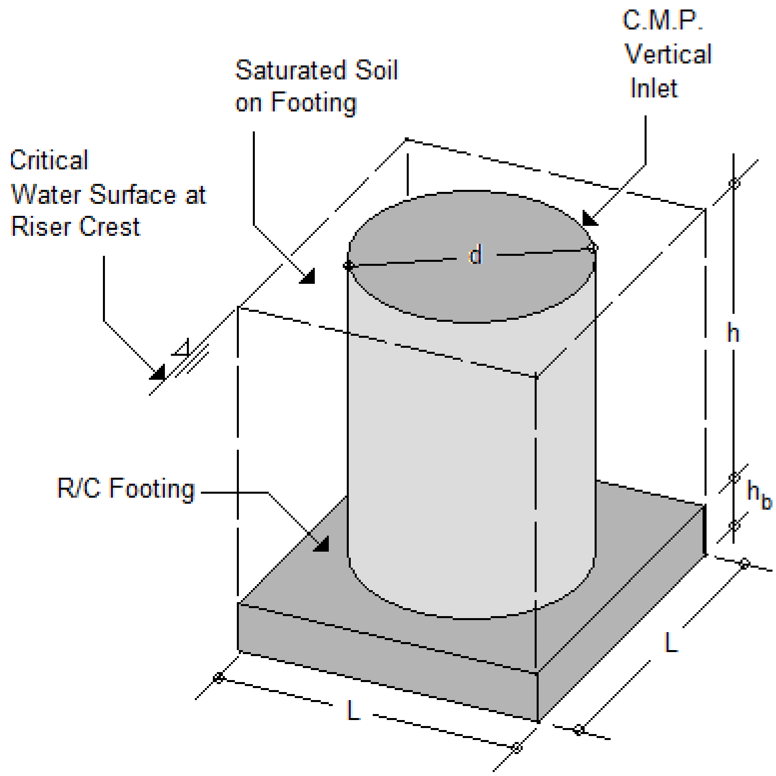
Example 6 – Buoyancy force calculation – flotation safety of CMP inlet

The figure below shows a cylindrical CMP (corrugated metal pipe) vertical inlet of diameter $d = 36\ in = 3\ ft$, supported on a square footing of side length L, and height $h_b = 13.5\ in = 1.125\ ft$ The inlet footing is covered with saturated soil up to a depth $h = 7\ ft$ which corresponds to the level of the inlet crest. The CMP inlet weighs $\omega_{CMP} = 38\ lb/ft$, the buoyant weight of saturated soil on the footing is $\omega_{SS} = 60\ lb/ft^3$, and the concrete in the footing weighs $\omega_C = 150\ lb/ft^3$. Determine the size L of footing required to make the CMP inlet safe from flotation with a factor of safety $F.S.= 1.5$. Assume that there is no frictional resistance between the inlet walls and the surrounding soil, and that no shear forces act on the outlet conduit. The minimum length L must be $L = d + 1ft$ to allow for placement of reinforcing steel bars outside of the inlet with *3 in* concrete cover and *2 in* clearance between bars and inlet.

The factor of safety, *F.S.* for this case, is defined as the ratio of the net downward forces (ΣF_d) to the net upward forces (ΣF_u):

$$F.S. = \frac{\Sigma F_d}{\Sigma F_u}$$

Here, Σ, indicates summation of forces. By requiring that the factor of safety be $F.S. = 1.5$, the downward forces are 50% larger than the upward forces ensuring that the inlet will be safe from flotation.



Using the minimum length of footing:

$$L = d + 1ft = 3\ ft + 1\ ft = 4\ ft$$

The downward forces in this problem include the weight of the inlet, W_{CMP}, the buoyant weight of saturated soil, W_{SS}, and the weight of footing, W_C. With the minimum length, these forces are calculated as follows:

$$W_{CMP} = \omega_{CMP}\,(h+h_b) = 38\ lb/ft \times (7\ ft + 1.125\ ft) = 308.75\ lb$$

$$W_{SS} = \omega_{SS}\,(L^2 h - \pi d^2/4\ h) = 60\ lb/ft^3 \times ((4\ ft)^2 \text{-} 3.1416 \times (3ft)^2/4) \times (7\ ft\,) = 3{,}751.19\ lb$$

$$W_C = \omega_C L^2 h_b = 150\ lb/ft^3 \times (4\ ft)^2 \times 1.125\ ft = 2{,}700\ lb$$

Thus,

$$\Sigma F_d = W_{CMP} + W_{SS} + W_C = 308.75\ lb + 3{,}751.19\ lb + 2{,}700\ lb = 6{,}759.94\ lb$$

The upward forces include the buoyancy forces on the riser, F_{BCMP}, and on the footing, F_{BC}, which are calculated as:

$$F_{BCMP} = \omega_w\,(\pi d^2/4)\,h = 62.4\ lb/ft^3 \times (3.1416 \times (3\ ft)^2/4) \times 7\ ft = 3{,}087.56\ lb$$

$$F_{BC} = \omega_w\,L^2 h_b = 62.4\ lb/ft^3 \times (4\ ft)^2 \times 1.125\ ft = 1{,}123.2\ lb$$

Thus,

$$\Sigma F_u = F_{BCMP} + F_{BC} = 3{,}087.56\ lb + 1{,}123.2\ lb = 4{,}210.76\ lb$$

The factor of safety for a side of footing $L = 4\ ft$ is:

$$F.S. = \frac{\Sigma F_d}{\Sigma F_u} = \frac{6{,}759.94\ \text{lb}}{4{,}210.76\ \text{lb}} = 1.61 > 1.5$$

Since the factor of safety calculated is larger than *1.5* the CMP inlet should be safe for buoyancy. If $F.S. < 1.5$, try a larger footing side length or height, and recalculate the sum of forces and the factor of safety until it is larger than the required value of 1.5.

0315 Hydrokinetics

Hydrokinetics is the study of fluids in motion. Motion in fluids is produced by the action of forces. For example, in pressurized pipelines pressure forces act as the driving forces in the flow, whereas, in open channel flow, it is the weight of water mass (gravity forces) that produces the motion. In both cases, friction between the water and the walls of the pipe or channel act as opposing forces. In steady flows, driving and opposing forces are in equilibrium.

The analysis of fluids in motion typically requires the determination of flow quantities such as discharge or velocity, or the determination of a linear quantity such as pipe diameter or flow depth in open channel flow. The determination of energy or head losses is also an important aspect of the analysis of fluids in motion. These analyses have practical applications in the operation of devices or systems through which fluids (water) flow, for example, irrigation pipes, outlet works from dams, and irrigation canals.

The principles of flow continuity and of conservation of energy are used to analyze fluids in motion.

0316 Flow Continuity

The equation of continuity represents a statement of conservation of mass in fluid flow. Consider, for example, a steady flow in a closed conduit of varying cross-section as shown in Figure 18 below. Let Q be the discharge through the conduit. The equation of continuity for steady flow (i.e., Q = constant) states that:

$$Q = V_1 A_1 = V_2 A_2 = V_3 A_3 \qquad \textbf{[Eq. 28]}$$

where V_1, V_2, *and* V_3 represent the flow velocities at sections 1, 2, and 3 respectively. The areas of the corresponding cross-sections are A_1, A_2, and A_3.

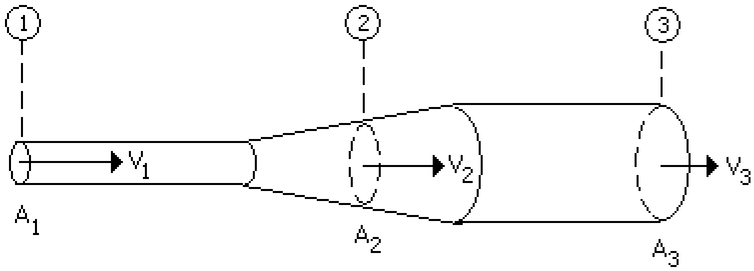


Figure 18. Flow continuity in a pipe expansion.

The cross-sectional area of a circular pipe of diameter *D* is given by:

$$A = \frac{\pi D^2}{4}$$ **[Eq. 29]**

And the discharge can be written as:

$$Q = VA = V \cdot \frac{\pi D^2}{4}$$ **[Eq. 30]**

Knowing the discharge *Q*, the velocity *V* in a circular conduit of diameter *D* can be calculated as:

$$V = \frac{4Q}{\pi D^2}$$ **[Eq. 31]**

In units of the English System, the velocity *V* is given in *ft/s* (or *fps*), while the discharge *Q* is given in ft^3/s or *cfs*. Using the International System, the velocity *V* is given in *m/s* and the discharge *Q* is given in m^3/s. Other units of discharge in the English System include *gal/min* (or *gpm*) and *gal/day* (or *gpd*), which are commonly used in water supply applications. In the operation of large reservoirs, for example, the use of *acre-foot/day* as a unit of discharge is not uncommon. In the International System one could also use m^3/min or m^3/day for large discharge operations, while the units of *liters/s* or *liters/min* are used for small discharges.

Example 7 – Equation of continuity in a pipe - discharge and velocity calculation

Suppose that in the pipeline of Figure 18 the diameters are $D_1 = 0.25\ ft$ (*3 in*), $D_2 = 0.50\ ft$ (*6 in*), and $D_3 = 0.75\ ft$ (*9 in*), and that the velocity at section 3 is measured to be $V_3 = 0.5\ fps$. Find the velocities in sections 1 and 2 and the discharge.

The discharge is calculated as:

$$Q = V_3 \cdot \frac{\pi D_3^2}{4} = 0.5\, ft/s \times \frac{3.1416 \times (0.75\, ft)^2}{4} = 0.22\, cfs$$

The velocities at sections 1 and 2 are, therefore, calculated as:

$$V_1 = \frac{4Q}{\pi D_1^2} = \frac{4 \times 0.22\, ft^3/s}{3.1416 \times (0.25\, ft)^2} = 4.48\ fps$$

and

$$V_2 = \frac{4Q}{\pi D_2^2} = \frac{4 \times 0.22\, ft^3/s}{3.1416 \times (0.5\, ft)^2} = 1.12\ fps$$

A branching pipeline in which a pipe of diameter D_1 splits into two pipelines with diameters D_2 and D_3 is shown below. Continuity requires that the discharges through sections 2 and 3 add to equal the discharge through section 1:

$$Q_1 = Q_2 + Q_3 \qquad \textbf{[Eq. 32]}$$

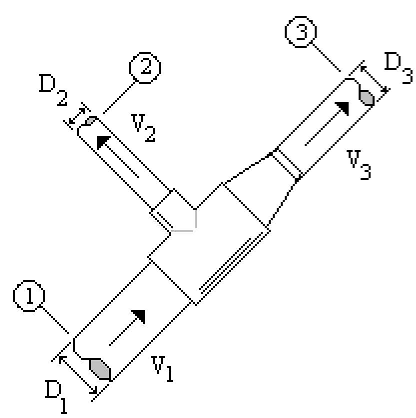


Figure 19. Schematic of flow in branching pipelines.

In terms of areas and velocities, the continuity equation for the branching pipe case is written as:

$$V_1 A_1 = V_2 A_2 + V_3 A_3 \qquad \textbf{[Eq. 33]}$$

Replacing the areas in terms of the diameters:

$$V_1 \cdot \frac{\pi D_1^2}{4} = V_2 \cdot \frac{\pi D_2^2}{4} + V_3 \cdot \frac{\pi D_3^2}{4} \qquad \textbf{[Eq. 34]}$$

Simplifying:

$$V_1D_1^2 = V_2D_2^2 + V_3D_3^2 \qquad \textbf{[Eq. 35]}$$

Example 8 – Equation of continuity – velocity and discharge calculation in branching pipeline
A *12-in*-diameter pipeline carrying water at a velocity of 3.5 *fps* branches into a *6-in*-diameter pipeline and a *9-in*-diameter pipeline. If the velocity in the *6-in* pipeline is measured to be *4.0 fps*, what is the velocity in the *9-in* pipeline, and what is the total discharge through the *12-in* pipeline?

Using $D_1 = 12\ in = 1\ ft$, $V_1 = 3.5\ fps$, $D_2 = 6\ in = 0.5\ ft$, $V_2 = 4.0\ fps$, and $D_3 = 9\ in = 0.75\ ft$, find V_3 and Q_1. From equation 35, i.e., $V_1D_1^2 = V_2D_2^2 + V_3D_3^2$, it follows that:

$$V_3 = \frac{V_1D_1^2 - V_2D_2^2}{D_3^2} = \frac{3.5\,fps \times (1\,ft)^2 - 4.0\,fps \times (0.5\,ft)^2}{(0.75\,ft)^2} = 4.44\,fps$$

The discharge through section 1 is:

$$Q_1 = V_1 \cdot \frac{\pi D_1^2}{4} = 3.5\,fps \times \frac{3.1416 \times (1\,ft)^2}{4} = 2.75\,cfs$$

The equation of continuity can also be applied to flow in open channels. Consider the flow in an open channel with a non-symmetric trapezoidal cross-section as illustrated in the figure below.

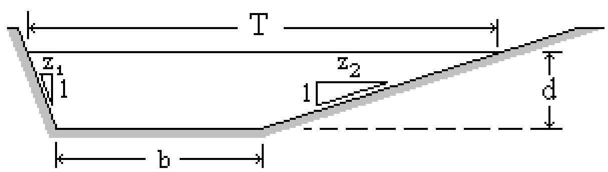


Figure 20. Non-symmetric trapezoidal cross-section in an open channel flow.

This cross-section is characterized by the *bottom width b*, the *side slopes* z_1 and z_2, and the *flow depth d*. The side slopes are interpreted as z_1 H:1V, i.e., z_1 units horizontal to 1 unit vertical. The *top width T* is the length of the free surface at the cross-section. For the non-symmetric trapezoidal cross-section shown above, the top width is given by

$$T = b + (z_1+z_2)d \qquad \textbf{[Eq. 36]}$$

and the area is calculated as the average of trapezoid's bases, *b* and *T*, multiplied by the height of the trapezoid, *d* :

$$A = \left(\frac{T+b}{2}\right) \cdot d = \left(b + \frac{z_1+z_2}{2}d\right) \cdot d \qquad \textbf{[Eq. 37]}$$

Example 9 – Equation of continuity – velocity in a trapezoidal open-channel cross-section

Suppose that the non-symmetric trapezoidal cross-section of Figure 20 has $b = 3\ ft$, $d = 1.3\ ft$, $z_1 = 0.5$, and $z_2 = 1$, and carries a discharge $Q = 15.2\ cfs$. The area is calculated as:

$$A = \left(b + \frac{z_1 + z_2}{2} d \right) \cdot d = \left(3\,ft + \frac{0.5 + 1}{2} \times 1.3\,ft \right) \times 1.3\,ft = 5.17\ ft^2$$

And the flow velocity is: $V = Q/A = 15.2\ cfs/5.17\ ft^2 = 2.94\ fps$

Example 10 – Equation of continuity – channel width reduction

The figure below shows a rectangular open channel that reduces in channel width from b_1 to b_2.

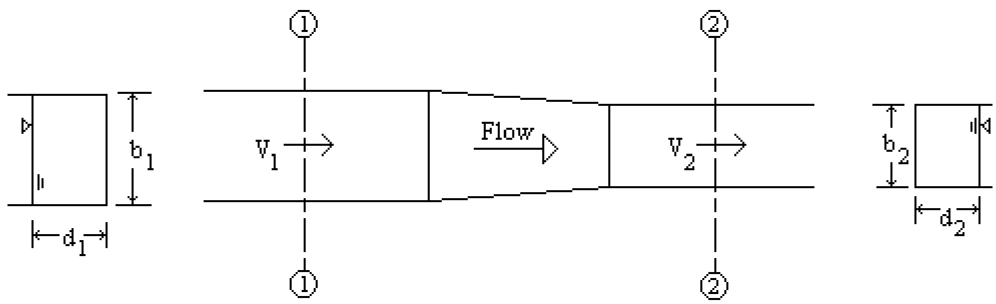


Figure 21. Contraction in a rectangular open channel.

In Figure 21 suppose that $b_1 = 7.5\ ft$, and $b_2 = 5.0\ ft$. The depth of flow and velocity in section (1) are $d_1 = 2.5\ ft$ and $V_1 = 4.5\ fps$. What depth of flow is required in section (2) in order to maintain the same flow velocity (i.e., $V_2 = V_1 = 4.5\ fps$)? What is the flow discharge through the channel?

With the cross-sectional shape being rectangular in both sections (1) and (2), the area of the cross-section is given by $A = bd$. Thus, the continuity equation (equation 28) can be written as:

$$Q = V_1 b_1 d_1 = V_2 b_2 d_2$$

The depth at section (2) is given by:

$$d_2 = \frac{V_1 b_1 d_1}{V_2 b_2} = \frac{4.5\,fps \times 7.5\,ft \times 2.5\,ft}{4.5\,fps \times 5.0\,ft} = 3.75\ ft$$

The discharge is calculated as:

$$Q = V_1 b_1 d_1 = 4.5\ fps \times 7.5\ ft \times 2.5\ ft = 84.38\ cfs$$

Example 11 – Equation of continuity – rectangular open channel diversion

Figure 22 shows a rectangular channel of width b_1 from which water is diverted through a lateral rectangular channel of width b_3. The section of the main channel downstream from the diversion has a width $b_2 = b_1$.

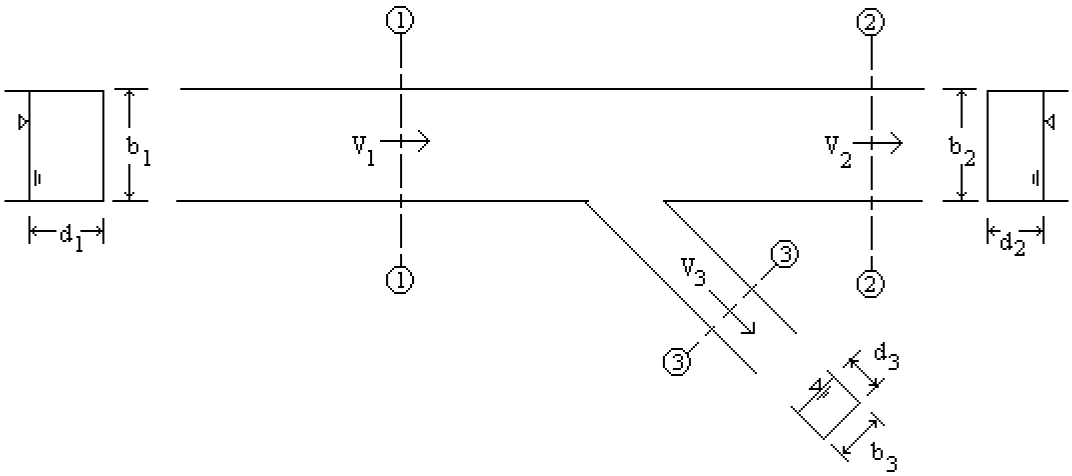


Figure 22. Rectangular open channel diversion.

Using $b_1 = b_2 = 10$ *ft*, $d_1 = 4.0$ *ft*, $V_1 = 1.2$ *fps*, $d_2 = 3.5$ *ft*, $b_3 = 5.0$ *ft*, $d_3 = 2.5$ *ft*, and $V_3 = 0.6$ *fps*, determine the flow velocity at section 2, V_2, as well as, the discharges through sections 1, 2, and 3.

For a branching channel as shown in Figure 22, continuity requires that $Q_1 = Q_2 + Q_3$, i.e., $V_1b_1d_1 = V_2b_2d_2 + V_3b_3d_3$. Thus, the velocity at section 2 is:

$$V_2 = (V_1b_1d_1 - V_3b_3d_3)/(b_2d_2) = (1.2\ fps\times 10\ ft\times 4\ ft - 0.6\ fps\times 5\ ft\times 2.5\ ft)/(10\ ft\times 3.5\ ft) =$$

$$V_2 = 1.157\ fps$$

The discharges through the three sections are calculated as follows:

$$Q_1 = V_1b_1d_1 = 1.2\ fps\times 10\ ft\times 4\ ft = 48\ cfs$$

$$Q_2 = V_2b_2d_2 = 1.157\ fps\times 10\ ft\times 3.5\ ft = 40.5\ cfs$$

$$Q_3 = V_3b_3d_3 = 0.6\ fps\times 5\ ft\times 2.5\ ft = 7.5\ cfs$$

0317 Conservation of Energy

In the analysis of fluid flow, three types of energy are typically considered: potential or elevation energy, pressure energy, and kinetic energy. Figure 23, below, illustrates these concepts using a simple reservoir-sprinkler system.

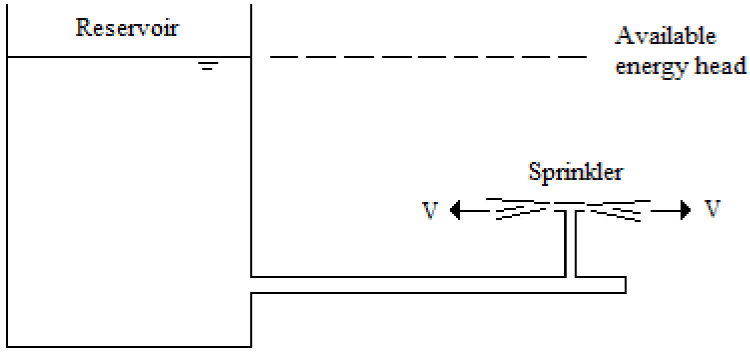


Figure 23. Flow energies illustrated with a simple reservoir-sprinkler system

The elevation of the water in the reservoir represents potential energy since it is this elevation that provides the available energy for the system. As the water flows through the pipe and is discharged through the sprinkler, it acquires motion which converts some of the potential energy into kinetic energy (energy of motion). Also, at the sprinkler, a measurable pressure exists that is related to pressure energy at that particular point.

In a flowing fluid system as the one illustrated above, energy is conserved as it is converted from one type of energy (potential) into another (pressure or kinetic). In fluid flow these energies are typically converted to energy heads that can be easily visualized as in the potential energy head of Figure 23. The description of these energy heads follows in the next sections.

0317.1 Potential Energy

Potential energy is the ability of a water mass to perform work because of the elevation of that mass of water with respect to an arbitrary datum line or reference level. A mass of weight *W*, at an elevation of *z* feet, has a potential energy equal to *Wz* (ft-lb) with respect to the datum. The elevation head, *z*, expresses not only a linear quantity (ft), but also energy per unit weight, i.e., *ft-lb/lb = ft.*

0317.2 Pressure Energy

Pressure energy at a point of a fluid flow is produced by the local pressure at the point. This pressure could be the result of a pumping action in a pipeline or of the weight of water above a certain point in open channel flow. The pressure head (pressure energy per unit weight) in pipeline flow is calculated as p/ω, pressure divided by the specific weight of water. In open channel flow applications, the pressure head is equal to the flow depth, *d*, and the pressure distribution is assumed to be hydrostatic.

0317.3 Kinetic Energy

The kinetic energy (K) of a mass (M) of fluid moving at a velocity (V) is given by K =1/2 MV^2. Since its weight is W = Mg, the kinetic energy per unit weight is calculated as:

$$h_v = \frac{K}{W} = \frac{\frac{1}{2}MV^2}{Mg} = \frac{V^2}{2g} \qquad \textbf{[Eq. 38]}$$

Where g is the acceleration of gravity ($g = 32.2\ ft/s^2$). The term h_v in equation 38 is referred to as the *velocity head*.

The *total energy head*, H, is the sum of the potential energy head (z), the pressure head (p/ω or d), and the velocity head ($V^2/2g$). All three forms of energy head may be expressed as a linear quantity (ft), or as energy per unit weight (ft-lb/lb).

Energy heads in pipe flow. Figure 24 illustrates the three different energy heads applied to pipe flow. Notice that, typically, the potential energy head (elevation) in a pipe flow is referenced at the centerline of the pipe. The pressure head is measured from the pipe centerline, and the distance from the datum to the top of the pressure head represents the piezometric head:

$$h = z + \frac{p}{\omega} \qquad \textbf{[Eq. 39]}$$

The difference between the total energy head and the piezometric represents the kinetic energy. The total energy in a pipe flow is given by:

$$H = h + \frac{V^2}{2g} = z + \frac{p}{\omega} + \frac{V^2}{2g} \qquad \textbf{[Eq. 40]}$$

The line representing the values of the total head, H, as function of position x along the pipeline is referred to as the *Energy Line* (E.L.), while that representing the values of the piezometric head, h, is referred to as the *Hydraulic Grade Line* (H.G.L.). Energy losses, h_f, are the losses due to friction between sections (1) and (2).

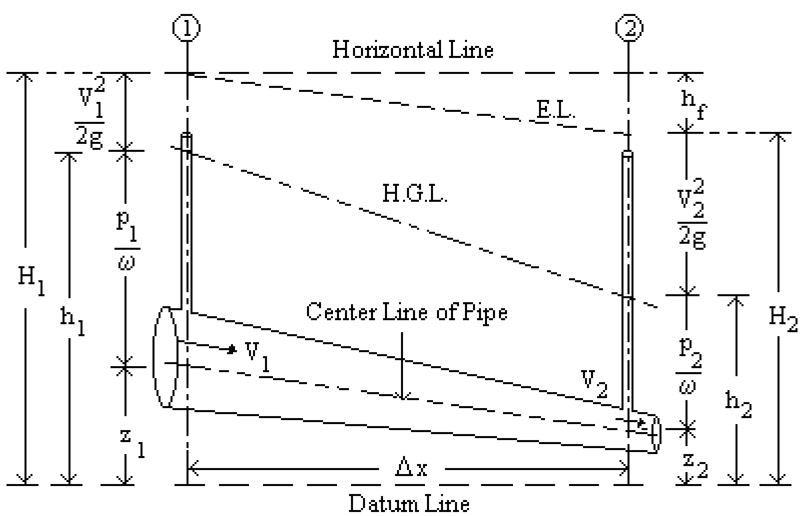


Figure 24. Energy heads in pipe flow.

Example 12 – Calculation of pressure, velocity, piezometric and total head

A manometer located at a point of a 6-*in*-diameter pipe shows a pressure reading of 6 *psi*. The point of pressure measurement is located at an elevation of 1500 *ft* above mean sea level and the pipe is carrying a discharge of 0.6 *cfs*. Calculate the pressure head, the velocity head, the piezometric head, and the total head.

The data given are D = 6 *in* = 0.5 *ft*, p = 6 *psi* = *6×144 psf* = 864 *psf*, z = 1500 *ft*, and Q = 0.6 *cfs*. Using ω = 62.4 lb/ft^3 for the specific weight of water, the pressure head is calculated as:

$$p/\omega = (864\ lb/ft^2)/(62.4\ lb/ft^3) = 13.85\ ft$$

The velocity can be calculated from the continuity principle using equation 31:

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 0.6\ ft^3/s}{3.1416 \times (0.5\ ft)^2} = 3.06\ fps$$

The velocity head is:

$$h_v = \frac{V^2}{2g} = \frac{(3.06\ ft/s)^2}{2 \times 32.2\ ft/s^2} = 0.15\ ft$$

The piezometric head is:

$$h = z + p/\omega = 1500\ ft + 13.85\ ft = 1513.85\ ft$$

The total head is

$$H = h + h_v = 1513.85\ ft + 0.15\ ft = 1514.0\ ft$$

Notice that in this example mean sea level is used as the datum line or reference level. If the pressure measurement point were located at 10 ft above the floor and the floor used as the datum line, then z = 10 *ft*, and the piezometric head and total head would be calculated as $h = z + p/\omega$ = 10 *ft* + 13.85 *ft* = 23.85 *ft*, and $H = h+h_v$ = *23.85 ft + 0.15 ft* = 24.0 *ft*.

In the previous example, the discharge Q was used to calculate the velocity V in a circular pipeline of diameter D. In terms of the discharge (see equations 31 and 38), the velocity head can be calculated as:

$$h_v = \frac{1}{2g} \cdot \left(\frac{4Q}{\pi D^2} \right)^2 = \frac{8Q^2}{\pi^2 g D^4} \qquad \textbf{[Eq. 41]}$$

Using this result the total energy head in a pipeline of diameter *D* carrying a discharge *Q* is given by:

$$H = z + \frac{p}{\omega} + \frac{8Q^2}{\pi^2 g D^4} \quad \textbf{[Eq. 42]}$$

Example 13 – Calculation of total head

A 2-*in* diameter pipeline carries a flow discharge of 0.3 *cfs*. At a point of the flow, located at an elevation of 2.5 *ft* above the floor, the pressure is measured to be 2 *psi*. What is the total energy head at that point?

Given *D* = 2 *in* = 2/12 *ft* = 0.166 *ft*, *Q* = 0.3 *cfs*, z = 2.5 *ft*, *p* = 2 *psi* = 2×144 *psf* = 288 *psf*, and, with ω = 62.4 lb/ft^3, the total head is:

$$H = z + \frac{p}{\omega} + \frac{8Q^2}{\pi^2 g D^4} = 2.5\,ft + \frac{288\,lb/ft^2}{62.4\,lb/ft^3} + \frac{8\times(0.3\,ft^3/s)^2}{3.1416^2\times 32.2\,ft/s^2\times(0.166\,ft)^4} = 10.1\,ft$$

Energy heads in open channel flow. The energy heads for open channel flow are illustrated in Figure 25. The elevation head, *z,* refers to the location of the channel bed, while the pressure head is represented by the flow depth *d*.

The sum of the potential energy head (*z*) and the flow depth (*d*) in open channel flow represents the water surface elevation or stage, *WS* = *z* + *d*. The difference between the total energy and the water surface elevation is the velocity head. The total energy in a channel flow is given by:

$$H = WS + V^2/2g = z + d + V^2/2g \quad \textbf{[Eq. 43]}$$

Of the total energy, the quantity called the *specific energy* is:

$$E = d + V^2/2g \quad \textbf{[Eq. 44]}$$

This quantity represents the flow energy measured with respect to the channel bed at a given cross-section.

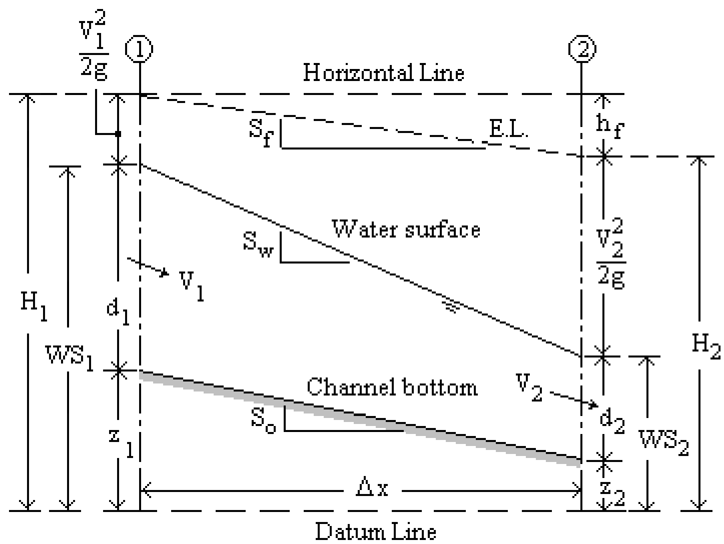


Figure 25. Energy heads in open channel flow.

In open channel flow, as illustrated in Figure 25, the hydraulic grade line is represented by the water surface. As in Figure 24, the term h_f represents the energy losses due to friction between section (1) and section (2).

In both the pipe flow and the open channel flow illustrated in Figure 24 and Figure 25, respectively, the horizontal distance between cross-sections 1 and 2 is referred to as Δx. The slope length of pipeline measured along its centerline, or the slope length of the channel between the two cross-sections, will be referred to as L.

Example 14 – Velocity head, specific energy, and total head in open-channel flow

A symmetric trapezoidal channel with bottom width b = 8.5 *ft* and side slope z = 0.5, carries a discharge of Q = 30 *cfs* at a depth d = 2.3 *ft*. Calculate the velocity head, and specific energy for this channel. If the channel bed is located at an elevation of 1255.32 *ft* above mean sea level, calculate the water surface elevation at that point, as well as the total energy head.

Equation 37, or methods in the following Section 0320, Open Channel Flow, can be used to calculate the cross-sectional area for a symmetric trapezoidal cross-section by taking $z_1 = z_2 = z$, i.e., $A = (b+z\cdot d)\cdot d$. For this example, the data given are b = 8.5 *ft*, z = 0.5, and d = 2.3 *ft*. Thus, A = (8.5 *ft* + 0.5×2.3 *ft*)×2.3 *ft* = 22.20 ft^2.

The flow velocity for this case is given by $V = Q/A$ = (30 ft^3/s)/ (22.20 ft^2) = 1.35 *ft/s*, and the velocity head is $h_v = V^2/(2g)$ = (1.35 $ft/s)^2$/ (2×32.2 ft/s^2) = 0.028 *ft*. Thus, the specific energy is: $E = d + h_v = d + V^2/(2g)$ = 2.3 *ft* + 0.028 *ft* = 2.328 *ft*.

The elevation of the channel bed is z = 1255.32 *ft* (notice that this is a different z than the side slope z for the cross-sectional geometry), and the water surface elevation is
$WS = z + d$ = 1255.32 *ft* + 2.3 *ft* = 1257.62 *ft*.

The total energy head is calculated as $H = z + E = 1255.32\ ft + 2.328\ ft = 1257.65\ ft$.

0317.4 Equation of Energy and Bernoulli's Principle

In the diagrams shown in Figure 24 and Figure 25, water flows from section (1) to section (2). The diagrams indicate that the total energy head at the upstream section (1) is larger than the total energy head at the downstream section (2), i.e., $H_1 > H_2$. The difference represents the *energy losses* h_f due to friction as the water moves from section (1) to section (2). The law of conservation of energy for both pipe flow and open channel flow can be written as:

$$H_1 = H_2 + h_f \qquad \textbf{[Eq. 45]}$$

where h_f represents the energy losses due to friction between sections (1) and (2).

Specifically, for the pipe flow case illustrated in Figure 24:

$$h_1 + \frac{V_1^2}{2g} = h_2 + \frac{V_2^2}{2g} + h_f \qquad \textbf{[Eq. 46]}$$

or

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f \qquad \textbf{[Eq. 47]}$$

The law of conservation of energy for open channel flow, as illustrated in Figure 25, is written as:

$$WS_1 + \frac{V_1^2}{2g} = WS_2 + \frac{V_2^2}{2g} + h_f \qquad \textbf{[Eq. 48]}$$

or

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f \qquad \textbf{[Eq. 49]}$$

In some instances of pipe flow (or other enclosed flow), the fluid can be assumed to be ideal and the friction losses are zero. This is an assumption of the *Bernoulli's principle*, and the equation of energy (Bernoulli's equation) can be written as:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} \qquad \textbf{[Eq. 50]}$$

In the most general case of pipe flow, however, friction losses, h_f, and local losses, h_L, (due to the presence of appurtenances in the pipe) must be included. Examples of appurtenances include elbows and valves. Taking into account both friction and local losses, the energy equation for pipelines can be written as:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f + h_L \qquad \textbf{[Eq. 51]}$$

Example 15 – Bernoulli's principle applied to sluice gate flow

The figure below shows a schematic of a sluice gate in a rectangular channel of width *b*. The energy line (E.L.) and the hydraulic grade line (H.G.L.) are also depicted. The horizontal energy line suggests no energy losses as the flow passes under the gate. Determine the discharge *Q* if the upstream and downstream depths are measured as 3.5 ft and 1.0 ft and the channel width, *b,* is 10 ft.

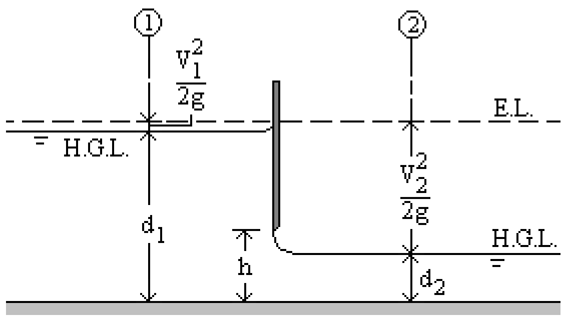


Figure 26. Sluice gate flow.

Since there are no energy losses and a horizontal bed, Bernoulli's equation reduces to:

$$d_1 + \frac{V_1^2}{2g} = d_2 + \frac{V_2^2}{2g}$$

Writing the velocity head in terms of the discharge, $V_1^2/2g = Q^2/(2gA_1^2)$ and $V_2^2/2g = Q^2/(2gA_2^2)$, and using $A_1 = bd_1$, $A_2 = bd_2$, Bernoulli's equation becomes:

$$d_1 + \frac{Q^2}{2gb^2d_1^2} = d_2 + \frac{Q^2}{2gb^2d_2^2}$$

Solving for *Q* and simplifying the result produces:

$$Q = bd_1d_2\sqrt{\frac{2g}{d_1 + d_2}}$$

$$Q = bd_1d_2\sqrt{\frac{2g}{d_1 + d_2}} = 10\,ft \times 3.5\,ft \times 1.0\,ft\sqrt{\frac{2 \times 32.2\,ft/s^2}{3.5\,ft + 1.0\,ft}} = 132.41 cfs$$

Example 16– Energy equation in pipelines

Let the diagram in Figure 24 represent a pipeline of constant diameter $D = 6\ in$ carrying a discharge $Q = 0.5\ cfs$. Let the elevations of points 1 and 2 be given by $z_1 = 7\ ft$ and $z_2 = 12.5\ ft$ with respect to an arbitrary horizontal reference level (datum line). The pressure at point 2 is measured to be $p_2 = 15.2\ psi$. Calculate the velocity head for this pipe flow. If the energy loss in the pipeline is estimated to be $h_f = 15\ ft$, what is the pressure at point 1, p_1 (*psi*)? Use $\omega = 62.4\ lb/ft^3$ for the specific weight of water.

Since the pipeline has a constant diameter, $D = 6\ in = 6/12\ ft = 0.5\ ft$, the velocities at points 1 and 2 are the same (equation 31):

$$V_1 = V_2 = V = 4Q/(\pi D^2) = 4\times(0.5\ ft^3/s)/(3.1416\times(0.5\ ft)^2) = 2.55\ fps$$

The velocity head throughout the pipeline is the same and equal to (equation 38):

$$h_v = V^2/2g = (2.55\ ft/s)^2/(2\times 32.2\ ft/s^2) = 0.10\ ft$$

Also, the velocity head can be calculated in terms of the discharge as (equation 41):

$$h_v = \frac{8Q^2}{\pi^2 g D^4} = \frac{8\times(0.5\,ft^3/s)^2}{3.1416^2\times 32.2\,ft/s^2\times(0.5\,ft)^4} = 0.10\,ft$$

The equation of energy for the case of Figure 24 is given by (equation 47):

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f$$

Since the velocities are the same ($V_1 = V_2$), the equation simplifies to:

$$z_1 + \frac{p_1}{\omega} = z_2 + \frac{p_2}{\omega} + h_f$$

From which the pressure head at point 1 is solved for, as:

$$p_1/\omega = z_2 - z_1 + p_2/\omega + h_f = 12.5\ ft - 7\ ft + (15.2\times 144\ lb/ft^2)/(62.4\ lb/ft^3) + 15\ ft$$

$$p_1/\omega = 55.58\ ft$$

Thus, the pressure at point 1 is:

$$p_1 = 55.58\ ft\times 62.4\ lb/ft^3 = 3468.19\ lb/ft^2 = 3468.19/144\ psi = 24.08\ psi$$

Example 17 - Energy equation in open channel flow

Suppose that Figure 25 represents a reach in an open channel flow with a rectangular cross-section of width b = 12.5 *ft*. The depth of flow at sections 1 and 2 are measured to be d_1 = 4.7 *ft* and d_2 = 3.8 *ft*, respectively, and the velocity at section 1 is V_1 = 4.2 *fps*. The channel bed elevations at sections 1 and 2 are given as z_1 = 25 *ft* and z_2 = 22.5 *ft*. Calculate the flow discharge Q and the energy loss, h_f, between sections 1 and 2.

The area at section 1 is $A_1 = bd_1 = 12.5\ ft \times 4.7\ ft = 58.75\ ft^2$, and the flow discharge is $Q = V_1A_1 = 4.2\ ft/s \times 58.75\ ft^2 = 246.75\ cfs$. The area at section 2 is $A_2 = bd_2 = 12.5\ ft \times 3.8\ ft = 47.50\ ft^2$, and the flow velocity at that location is $V_2 = Q/A_2 = (246.75\ ft^3/s) / (47.50\ ft^2) = 5.19\ fps$.

The energy equation for the case of Figure 25 is written as (equation 49):

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f$$

From which:

$$h_f = (z_1 + d_1 + V_1^2/2g) - (z_2 + d_2 + V_2^2/2g) =$$

$$= [(25\ ft + 4.7\ ft + (4.2\ \mathrm{ft/s})^2/(2\times 32.2\ \mathrm{ft/s^2})] - [(22.5\ ft + 3.8\ ft + (5.19\ \mathrm{ft/s})^2/(2\times 32.2\ \mathrm{ft/s^2})]$$

$$= 3.26\ ft$$

0317.5 Hydraulic and Energy Gradients

Figure 24 and Figure 25 represent the variation of the different energy heads in pipe flow and open channel flow, respectively. In both types of flow the line joining the total energy heads along the pipeline or open channel is the *energy line* (E.L.). In pipe flow, the line joining the piezometric heads along the pipeline ($h = z + p/\omega$) is the *hydraulic grade line* (H.G.L.), whereas, in open-channel flow the water surface represents the hydraulic grade line.

For both pipe flow and open channel flow the *energy gradient*, or *slope of the energy line*, S_f, is defined as the rate of friction head loss, h_f, per unit length, L, of the pipeline or open channel:

$$S_f = \frac{h_f}{L} = \frac{H_1 - H_2}{L} \qquad \textbf{[Eq. 52]}$$

However, in open channel flow, the channel bed slope, S_o, is typically very small such that the length of the channel, L, is approximately equal to the horizontal distance Δx, i.e., $L \approx \Delta x$. The *energy gradient for open channels*, therefore, can be defined as:

$$S_f \approx \frac{h_f}{\Delta x} = \frac{H_1 - H_2}{\Delta x} \qquad \textbf{[Eq. 53]}$$

The *hydraulic gradient* is the *slope of the hydraulic grade line*. For pipe flow, such gradient is defined as the change in piezometric head per unit length of pipe:

$$S_h = \frac{h_1 - h_2}{L} \qquad \textbf{[Eq. 54]}$$

Whereas, for an open channel flow the hydraulic gradient is the *water surface slope*. This slope is the change in water surface elevation, *WS*, per unit of horizontal distance along the channel path:

$$S_w = \frac{WS_1 - WS_2}{\Delta x} \qquad \textbf{[Eq. 55]}$$

Example 18– Hydraulic and energy gradients in pipe flow

The diameter of a 10-*ft*-long pipeline tapers from 1-*ft*-diameter at section 1 to 0.5-*ft*-diameter at section 2 of Figure 24. Let the pressure at section 1 be p_1 = 6.0 psi and that at section 2 be p_2 = 5.5 *psi*. The pipeline is laid so that point 1 is at elevation z_1 = 12.5 *ft* and z_2 = 6.2 *ft*. If the pipeline is carrying a flow Q = 1.5 *cfs*, determine the friction loss, h_f, the hydraulic gradient, S_h, and the energy gradient, S_f, for the flow.

For L = 10 *ft*, D_1 = 1 *ft*, D_2 = 0.5 *ft*, p_1 = 6.0 *psi*, p_2 = 5.5 *psi*, z_1 = 12.5 *ft*, and z_2 = 6.2 *ft*, the velocities are:

$$V_1 = 4Q/(\pi D_1^2) = (4\times 1.5\ ft^3/s)/(3.1416\times(1 ft)^2) = 1.91\ fps$$

and

$$V_2 = 4Q/(\pi D_2^2) = (4\times 1.5\ ft^3/s)/(3.1416\times(0.5\ ft)^2) = 7.64\ fps$$

The piezometric or pressure heads at sections 1 and 2 are:

$$h_1 = z_1 + \frac{p_1}{\omega} = 12.5\,ft + \frac{6.0\times 144\ lb/ft^2}{62.4\ lb/ft^3} = 26.35\,ft$$

and

$$h_2 = z_2 + \frac{p_2}{\omega} = 6.2\,ft + \frac{5.5\times 144\ lb/ft^2}{62.4\ lb/ft^3} = 18.89\,ft$$

The difference in piezometric heads is:

$$\Delta h = h_1 - h_2 = 26.35\ ft - 18.89\ ft = 7.46\ ft$$

The total energy heads at sections 1 and 2 are:

$$H_1 = h_1 + \frac{V_1^2}{2g} = 26.35\,ft + \frac{(1.91\,ft/s)^2}{2 \times 32.2\,ft/s^2} = 26.41\,ft$$

and

$$H_2 = h_2 + \frac{V_2^2}{2g} = 18.89\,ft + \frac{(7.64\,ft/s)^2}{2 \times 32.2\,ft/s^2} = 19.79\,ft$$

Thus, the energy loss is:

$$h_f = H_1 - H_2 = 26.41\,ft - 19.79\,ft = 6.62\,ft$$

The hydraulic gradient is calculated as:

$$S_h = \frac{\Delta h}{L} = \frac{7.46\,ft}{10\,ft} = 0.746$$

The energy gradient is:

$$S_f = \frac{h_f}{L} = \frac{6.62\,ft}{10\,ft} = 0.662$$

Example 19 - Hydraulic and energy gradients in open channel flow

In example 17, utilizing the energy equation in open channel flow, the following flow parameters were given or calculated: rectangular cross-section of width b = 12.5 *ft*. Depths of flow: d_1 = 4.7 *ft* and d_2 = 3.8 *ft*. Flow velocities: V_1 = 4.2 *fps* and V_2 = 5.19 *fps*. Bed elevations: z_1 = 25 *ft* and z_2 = 22.5 *ft*. Energy head loss, h_f = 3.26 *ft*.

If the distance between the two sections is Δx = 2500 ft, calculate the energy gradient and the hydraulic gradient for this flow.

The energy gradient:

$$S_f = h_f / \Delta x = 3.26\ ft/2500\ ft = 0.0013$$

The water surface elevations at sections 1 and 2 are, respectively, $WS_1 = z_1 + d_1 = 25\ ft + 4.7\ ft = 29.7\ ft$, and $WS_2 = z_2 + d_2 = 22.5\ ft + 3.8\ ft = 26.3\ ft$. Thus, the hydraulic gradient, or water surface slope, is:

$$S_w = (WS_2 - WS_1)/\Delta x = (29.7\ ft - 26.3\ ft)/2500\ ft = 3.4\ ft/2500\ ft = 0.00136$$

0320 Open Channel Flow

Open channel flow occurs when water is conveyed to a lower elevation through a conduit or channel open to the atmosphere or when a pipe flows without being full. Open channel flow is also referred to as free-surface flow. Flows in creeks, rivers, aqueducts, flumes, irrigation canals, gutters, and culverts are examples of open channel flows.

Open channels occur on a slope. If the slope is in the direction of the flow it is referred to as a favorable slope. If the slope is opposite to the direction of the flow, then the slope is referred to as an adverse slope. An open channel could also have a horizontal bed, in which case the slope is zero. A channel of constant slope and constant cross-section which does not change its alignment is referred to as a *prismatic channel*. Such is often the case for constructed channels. Natural channels, on the other hand, are often highly irregular showing varying alignment, curves, and changing cross-sectional geometry.

0321 Uniform Open Channel Flow

Uniform flow in a prismatic open channel occurs when the flow depth remains constant for a constant discharge. Uniform flow typically develops in long prismatic channels, away from head or tail sections. Natural channels rarely maintain uniform flow for long reaches.

Figure 27 shows the forces acting on a section of length *L* of uniform flow on a channel laid on a slope $S_o = tan(\theta_o)$. The slope is sufficiently small so that the distribution of pressure with depth in the flow is hydrostatic, and so that $S_o = tan(\theta_o) \approx sin(\theta_o) \approx \theta_o$ (measured in radians).

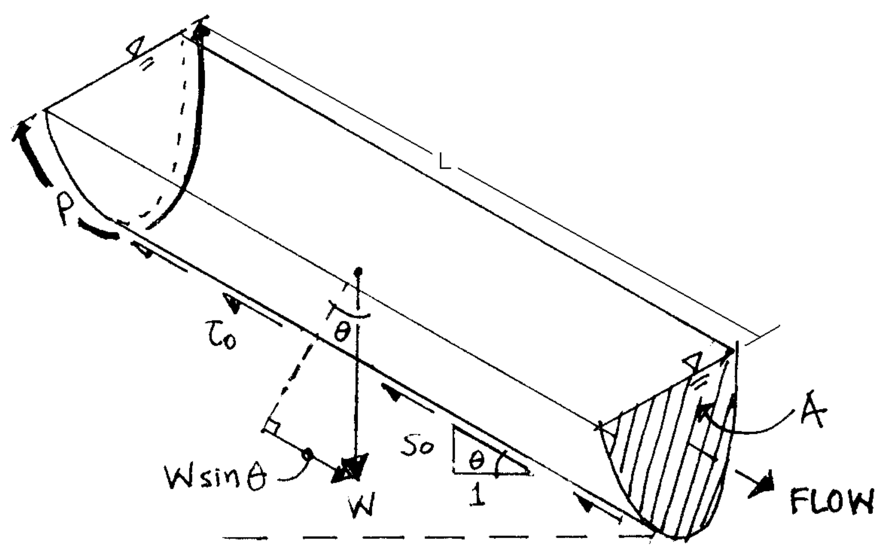


Figure 27. Schematic of uniform flow in open channels.

Because the flow depth and the cross-sectional shapes are the same at both ends of the flow element shown, the pressure forces acting at the upstream and downstream cross-sections of the flow element will cancel each other. The remaining driving force will be the component of the weight of the flow element parallel to the channel bed. This force is given by:

$$F_D = W \cdot \sin(\theta_o) = \omega \cdot A \cdot L \cdot \sin(\theta_o) \approx \omega \cdot A \cdot L \cdot S_o$$

Where *A* is the cross-sectional area of the flow.

Opposing this driving force is an opposite force due to friction on the channel walls, also known as the shear force. In Figure 27, τ_o represents the bed shear stress (assumed uniform through the channel), and the total shear force is given by:

$$F_S = \tau_o \cdot L \cdot P$$

Where *P*, the *wetted perimeter*, is the length of the channel cross-section in contact with the water.

For a uniform flow to occur, the driving force (weight component) and the shear force must be in equilibrium, i.e., $F_S = F_D$, or:

$$\tau_o \cdot L \cdot P = \omega \cdot A \cdot L \cdot S_o$$

The bed shear stress is, therefore, given by:

$$\tau_o = \omega \cdot \frac{A}{P} \cdot S_o = \omega \cdot R \cdot S_o \quad \textbf{[Eq. 56]}$$

Where *R*, the *hydraulic radius*, has been introduced:

$$R = A/P \quad \textbf{[Eq. 57]}$$

Example 20 – Shear stress in uniform open-channel flow

Calculate the shear stress on the bed of an open channel with a hydraulic radius, $R = 0.75$ *ft*, laid on a slope, $S_o = 0.001$. Use $\omega = 62.4$ lb/ft^3 for the specific weight of water.

The shear stress is calculated as:

$$\tau_o = \omega \cdot R \cdot S_o = 62.4 \frac{lb}{ft^3} \times 0.75\ ft \times 0.001 = 0.0468 \frac{lb}{ft^2} =$$

$$= 0.0468 \times 144 \frac{lb}{in^2} = 6.74\ psi$$

The shear stress τ_o can be written in terms of the mean flow velocity V, the density of water ρ, and a dimensionless "drag" coefficient C_D:

$$\tau_o = \frac{1}{2} C_D \cdot \rho \cdot V^2$$

Substituting this result into equation 56 for τ_o, and using $\omega = \rho \cdot g$:

$$\frac{1}{2} C_D \cdot \rho \cdot V^2 = \rho \cdot g \cdot R \cdot S_o$$

From which it follows that the velocity can be calculated as:

$$V = \sqrt{\frac{2g}{C_D}} \cdot \sqrt{R \cdot S_o} = C \cdot \sqrt{R \cdot S_o} \qquad \textbf{[Eq. 58]}$$

This result is known as Chezy's equation, and the coefficient C is referred to as Chezy's coefficient. Typical values of the Chezy coefficient range between 80 and 140. For example, the value C = 120 is typically used for concrete.

Example 21 – Velocity calculation in open-channel flow using Chezy's equation
Using the Chezy equation with C = 120, calculate the flow velocity in an open channel with a hydraulic radius $R = 0.75\ ft$, laid on a slope $S_o = 0.001$.

The velocity in the open channel flow is calculated as:

$$V = C \cdot \sqrt{R \cdot S_o} = 120 \times \sqrt{0.75 \times 0.001} = 3.29\ fps$$

While Chezy's equation is dimensionally sound, a different equation, Manning's equation, has been used for more than a century for solving practical problems of uniform flow in open channels. Manning's equation has become the most widely used uniform flow equation, with many references available for the selection of a Manning's coefficient. For historical notes on the development of both the Chezy's and Manning's equations, refer to Chow (1959). Manning's equation is presented in section 0321.2.

0321.1 Geometric Characteristics of Prismatic Channels

In general, we are interested in calculating the following geometric characteristics of open channel cross-sections:

- The cross-sectional area, A (ft^2)
- The wetted perimeter, P (ft)
- The top-width of the section, T (ft)

- The hydraulic radius, R = A/P (ft)
- The hydraulic depth, D_h = A/T (ft)

The cross-sectional area, wetted perimeter, and hydraulic radius have been defined in section 0321. The top-width of the section (T) is the length of the water surface in the cross-section, whereas the hydraulic depth (D_h) is the depth of an equivalent rectangular cross-section of the same area and width equal to the top-width (T).

The calculation of geometric characteristics for open channels of regular-shaped cross-sections is straightforward as indicated next.

<u>Non-symmetric trapezoidal channel</u>
The figure below shows a non-symmetric trapezoidal channel cross-section.

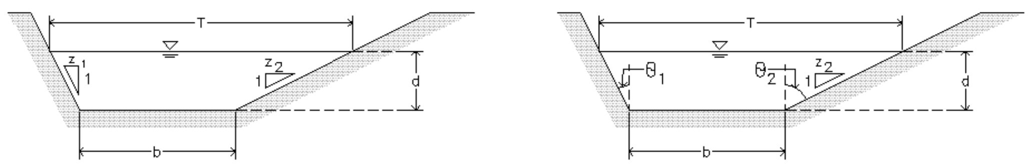


Figure 28. Non-symmetric trapezoidal channel

Given the bottom width of the cross-section, b, the side slopes z_1 and z_2, and the flow depth, d, the geometric characteristics of the non-symmetric trapezoidal cross-section are:

$$A = \frac{1}{2}\,(2b + (z_1+z_2)\,d)\,d, \quad P = b + d\,\left(\sqrt{1+z_1^2} + \sqrt{1+z_2^2}\right), \quad T = b + (z_1+z_2)\,d \qquad \textbf{[Eq. 59]}$$

In this case, z_1 and z_2 represent the dimensionless side slope as horizontal displacement per unit vertical rise. For example, on the left-hand side the channel bank slopes z_1 ft horizontal per each vertical foot. This is also represented as z_1H:*1*V (z_1 ft horizontal to 1 ft vertical). The right-hand-side slope would be represented as z_2H:*1*V. If the angles θ_1 and θ_2 are given, the side slopes can be calculated as:

$$z_1 = \tan(\theta_1), \quad z_2 = \tan(\theta_2) \qquad \textbf{[Eq. 60]}$$

Example 22 – Geometric characteristics of a non-symmetric trapezoidal cross-section
A non-symmetric trapezoidal channel flowing at a depth d = 1.5 ft, has a bottom width b = 6.5 ft, and side slopes laid on angles, $\theta_1 = 30^o$ and $\theta_2 = 60^o$, with respect to a vertical line. Calculate the side slopes and the geometric characteristics for this cross-section.

First, the side slopes, z_1 and z_2 are calculated as:

$$z_1 = \tan(\theta_1) = \tan 30^o = 0.5773$$

$$z_2 = \tan(\theta_2) = \tan 60^{o} = 1.7320$$

The geometric characteristics of this cross-section are calculated as follows:

$$A = ½ (2b + (z_1+z_2)\, d)\, d = ½ \times (2 \text{ x } 6.5 \text{ ft} + (0.5773+1.7320) \times 1.5 \text{ ft}) \times 1.5 \text{ ft} = 12.35 \text{ ft}^2$$

$$P = b + d(\sqrt{1+z_1^2} + \sqrt{1+z_2^2}\,) = 6.5 \text{ ft} + 1.5 \text{ ft} \times (\sqrt{1+0.5773^2} + \sqrt{1+1.7320^2}\,) = 11.23 \text{ ft}$$

$$R = A/P = 12.35 \text{ ft}^2/11.23 \text{ ft} = 1.10 \text{ ft}$$

$$T = b+(z_1+z_2)d = 6.5 \text{ ft} + (0.5773+1.7320) \times 1.5 \text{ ft} = 9.96 \text{ ft}$$

$$D_h = A/T = 12.35 \text{ ft}^2/9.96 \text{ ft} = 1.24 \text{ ft}$$

The figure below shows different cross-sections that can be derived from the non-symmetric trapezoidal cross-section. The geometric elements of some of them are presented in detail below.

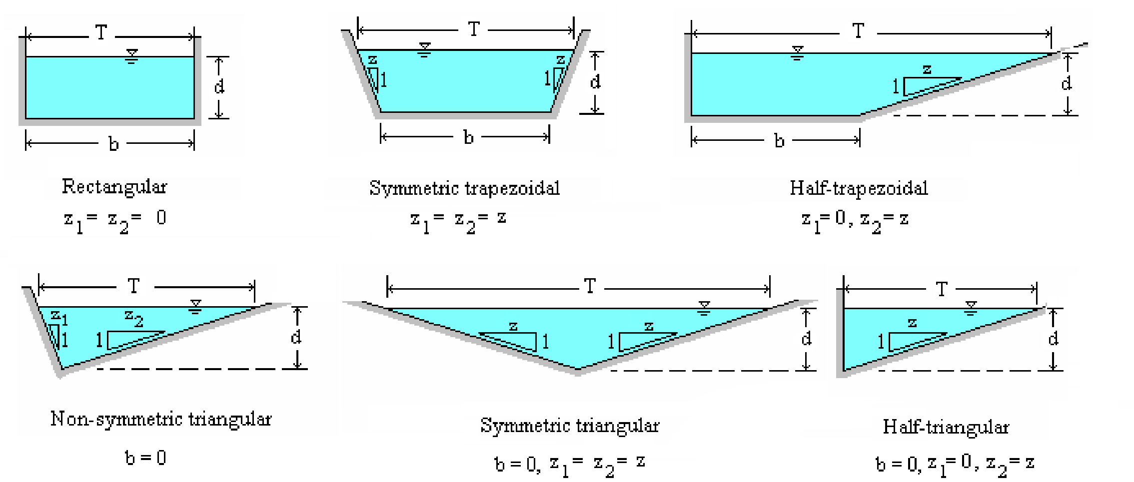


Figure 29. Channel cross-sections that can be derived from a non-symmetric trapezoidal cross-section.

Wide channels

Channels of approximately rectangular shape that are much wider than they are deep, say, b/d > 10, are referred to as wide channels. For such channels, the hydraulic radius is approximately equal to the channel depth and to the hydraulic depth:

$$R \approx d \approx D_h \qquad \textbf{[Eq. 61]}$$

Example 23 – Hydraulic radius for wide rectangular channel

For a rectangular channel with b = 20 ft and d = 1.5 ft, calculate the hydraulic radius using the full formula and the approximation for a wide channel.

The hydraulic radius using the full definition for a rectangular channel is:

$$R_R = bd/(b+2d) = (20 \text{ ft}) \times (1.5 \text{ ft})/(20 \text{ ft} + 2 \times 1.5 \text{ ft}) = 1.3 \text{ ft}$$

Using the wide-channel approximation, $R_W = d = 1.5$ ft, results in about a 15% difference.

Symmetric trapezoidal channel

Many constructed trapezoidal channels are symmetric, thus, $z_1 = z_2 = z$, and the geometric characteristics are calculated as:

$$A = (b+zd)\,d, \quad P = b + 2d\sqrt{1+z^2}, \quad T = b+2zd \qquad \textbf{[Eq. 62]}$$

Example 24 – Geometric characteristics of a symmetric trapezoidal cross-section in open channels

A symmetric trapezoidal open channel with a bottom width b = 2.5 ft and side slope 1.5 H: 1 V (z = 1.5) flows with a water depth d = 0.75 ft. Determine the geometric characteristics for this open channel. Also, determine the angle θ that the channel banks make with a vertical line.

The solution is given by:

$$A = (b+zd)d = (2.5 \text{ ft} + 1.5\times0.75 \text{ ft}) \times 0.75 \text{ ft} = 2.72 \text{ ft}^2$$

$$P = b + 2d\sqrt{1+z^2} = 2.5 \text{ ft} + 2 \times 0.75 \text{ ft} \times \sqrt{1+1.5^2} = 5.20 \text{ ft}$$

$$R = A/P = 2.72 \text{ ft}^2/5.20 \text{ ft} = 0.52 \text{ ft}$$

$$T = b+2zd = 2.5 \text{ ft} + 2 \times 1.5 \times 0.75 \text{ ft} = 4.75 \text{ ft}$$

$$D_h = A/T = 2.72 \text{ ft}^2/4.75 \text{ ft} = 0.57 \text{ ft}$$

To calculate the angle θ that the channel banks make with a vertical line, use z = tan (θ), or:

$$\theta = \tan^{-1}(z) = \tan^{-1}(1.5) = 56.31^o$$

Circular channel

A circular open channel results when water flows in a pipe or circular conduit with a free surface, as shown below.

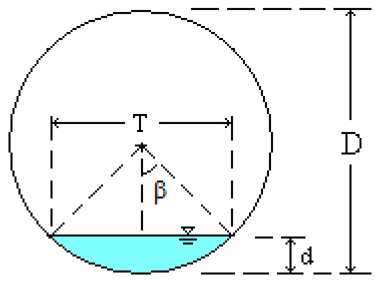


Figure 30. Circular cross-section in open channel flow.

For such a cross-section, the diameter D and the flow depth d (with d<D) are typically known. To calculate the geometric characteristics, the central angle β (in radians) is calculated as:

$$\beta = \cos^{-1}\left(1 - 2\frac{d}{D}\right)$$ **[Eq. 63]**

The geometric characteristics are calculated as:

$$A = \frac{D^2}{4}\left(\beta - \sin(\beta)\cdot\cos(\beta)\right)$$ **[Eq. 64]**

$$P = \beta D$$ **[Eq. 65]**

$$T = D \cdot \sin(\beta)$$ **[Eq. 66]**

Note: A full circle is composed of 360^o or 2π (= 6.2832) radians. The relationship between angles in radians (θ^r) and angles in degrees (θ^o) is the same as the ratio of $2\pi:360 = \pi:180$, or:

$$\frac{\theta^r}{\theta^o} = \frac{\pi}{180} = 0.01745$$ **[Eq. 67]**

$$\frac{\theta^o}{\theta^r} = \frac{180}{\pi} = 57.29$$ **[Eq. 68]**

Example 25 – Geometric characteristics of a circular cross-section in open channels

A pipeline 2.5-ft in diameter is flowing at a depth of 6 inches. Determine the geometric characteristics of this cross-section.

For D = 2.5 ft and d = 6 in = 6/12 = 0.5 ft, the central angle β is calculated as follows:

$$\beta = \cos^{-1}\left(1 - 2\frac{d}{D}\right) = \cos^{-1}\left(1 - 2\times\frac{0.5\,ft}{2.5\,ft}\right) = \cos^{-1}(0.6) = 0.9273^{r}$$

The corresponding geometric characteristics are as follows:

$$A = \frac{D^2}{4}\left(\beta - \sin(\beta)\cdot\cos(\beta)\right) = \frac{(2.5\,ft)^2}{4}\times\left(0.9273 - \sin(0.9273)\times\cos(0.9273)\right) = 0.70\ ft^2$$

$$P = \beta D = 0.9273 \times 2.5\ ft = 2.32\ ft$$

$$R = A/P = 0.70\ ft^2/2.32\ ft = 0.30\ ft$$

$$T = D\cdot\sin(\beta) = 2.5\ ft \times \sin(0.9273) = 2\ ft$$

Parabolic channel

The parabolic cross-section is used to approximate some natural cross-sections. Some waterways are also constructed of parabolic shape. A parabolic cross-section is characterized by its top width T and its depth d, as shown in the figure below.

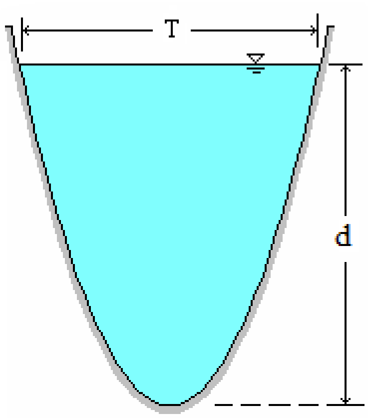


Figure 31. Parabolic cross-section

The geometric characteristics of this cross section are calculated as follows:

$$A = \frac{2}{3}\cdot d\cdot T \qquad \textbf{[Eq. 69]}$$

$$P = \frac{T}{2}\sqrt{1+\left(\frac{4\cdot d}{T}\right)^2} + \frac{T^2}{8\cdot d}\cdot \ln\left(\frac{4\cdot d}{T} + \sqrt{1+\left(\frac{4\cdot d}{T}\right)^2}\right) \qquad \textbf{[Eq. 70]}$$

Example 26 – Geometric characteristics of a parabolic cross-section
A parabolic waterway has a depth d = 2 ft and a top width T = 50 ft. Determine the geometric characteristics of the cross-section.

$$A = \frac{2}{3}\cdot d\cdot T = \frac{2}{3}\times 2\ ft \times 50\ ft = 66.67\ ft^2$$

$$P = \frac{T}{2}\sqrt{1+\left(\frac{4\cdot d}{T}\right)^2} + \frac{T^2}{8\cdot d}\cdot \ln\left(\frac{4\cdot d}{T} + \sqrt{1+\left(\frac{4\cdot d}{T}\right)^2}\right) = \frac{50\ ft}{2}\times 1.013 + \frac{(50\ ft)^2}{8\times 2\ ft}\times \ln(0.16+1.013)$$

$$P = 50.26\ ft$$

$$R = \frac{A}{P} = \frac{66.67\ ft^2}{50.26\ ft} = 1.33\ ft$$

$$D_h = \frac{A}{T} = \frac{66.67\ ft^2}{50\ ft} = 1.33\ ft$$

0321.2 Manning's Equation

The widely-used Manning's equation was introduced earlier as an example of a non-dimensionally homogeneous equation. The equation, named after the Irish engineer who proposed it in the late 1800's, is given by:

$$V = \frac{C_u}{n} R^{2/3} S_0^{1/2} \qquad \textbf{[Eq. 71]}$$

where *V* is the flow velocity, C_u is a constant that depends on the system of units used ($C_u = 1.0$ in the S.I., and $C_u = 1.486$ in the *E.S.*), *R* is the hydraulic radius, S_o is the channel bed longitudinal slope, and *n* is the Manning's resistance coefficient. Manning's *n-values* are available from a variety of sources; some values are provided in the next section, Table 2. Manning's resistance coefficients for open channel flow.

Many times, it is preferable to write the Manning's equation in terms of the water discharge Q = VA:

$$Q = V \cdot A = \frac{C_u}{n} AR^{2/3} S_0^{1/2} \quad \textbf{[Eq. 72]}$$

Since the hydraulic radius is defined as R = A/P, the Manning's equation can also be written as:

$$Q = \frac{C_u}{n} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2} \quad \textbf{[Eq. 73]}$$

The values of A and P depend on the cross-sectional geometry and the flow depth. If A and P are known, calculation of the discharge Q (see above), Manning's n, or the bed slope S_o, is straightforward:

$$n = \frac{C_u}{Q} \frac{A^{5/3}}{P^{2/3}} S_0^{1/2} \quad \textbf{[Eq. 74]}$$

and

$$S_o = \left(\frac{Q \cdot n}{C_u} \right)^2 \cdot \frac{P^{4/3}}{A^{10/3}} \quad \textbf{[Eq. 75]}$$

The *USDA-NRCS Hydraulics Formula* program allows for the calculation of the discharge *Q* for a variety of cross-sections. Examples are provided below. (See section *0321.4*).

Calculations involving the geometric parameters (e.g., flow depth, channel width) are more complicated because the geometric characteristics A and P are raised to fractional powers (5/3, 2/3), and because the geometric characteristics are a function of the parameters. Solving the nonlinear equations for any of the geometric parameters typically requires an iterative procedure. The calculations are best done with a spreadsheet, which uses a numerical analysis method to automate the iterative procedure.

Although the Darcy-Weisbach equation was originally developed for pipe flow (see section 0331.2, Darcy-Weisbach Equation and Friction Factor), it has been adapted for open channel flow. Refer to NEH 654.0609(d) for information on applying the Darcy-Weisbach equation to open channel flow.

0321.3 Manning's Resistance Coefficient

Manning's resistance coefficient, *n* (see equation 71), is assumed to be a dimensionless number in modern-day practice so that the same n-value can be used in both the English and International Systems of units. The n-value depends on a number of factors including:

- Roughness of the channel lining
- Changes in channel alignment

- Changes in cross-section geometry
- Presence of obstructions
- Presence of vegetation
- Water depth

The selection of Manning's n depends heavily on the practitioner's experience. Photos and descriptions of channels and floodplains for which n-values have been calculated, based on measured discharges and high water marks, are useful (See Barnes, 1967, or Fasken, 1963). Photo galleries of n-values have been established online.

When an n-value must be selected for a channel outside a practitioner's experience, a more regimented approach is sometimes helpful (See Cowan, 1956; this method is also presented in Chow, 1959 and Fasken, 1963). In Cowan's approach, n-values are selected and summed for five channel conditions:

- Bed material
- Degree of surface irregularity
- Variation of channel cross section
- Effect of obstructions
- Vegetation

The summed n-value may be further adjusted based on the degree of channel meandering.

After selection, n-values should be calibrated with any available high water marks and gaged flow data. In the calibration process, Manning's n-values should be reasonably adjusted to match observed water surface profiles.

The retardance potential of a grass-lined open channel, including vegetated earthen spillways, can be better evaluated with a retardance curve index rather than Manning's n (See Temple, et.al., 1987). The retardance curve index is based on the grass stem length and stem density.

The following table shows typical values of Manning's resistance coefficients for constructed channels lined with different materials, as well as for natural streams, excavated earth channels, and floodplains. The values were compiled from many of the following references: Barnes (1967), Cowan (1956), Chaudhry (1993), Chow (1959), Fasken (1963), French (1985), Mays (1999), Munson et al. (1998), Streeter et. al. (1998), and Temple et al. (1987).

Table 2. Manning's resistance coefficients for open channel flow

Surface lining	*Manning's n-value*	*Surface lining*	*Manning's n-value*
Concrete, finished	0.012	Clean, straight, natural streams	0.030
Concrete, unfinished	0.014	**Excavated earth channels**	
Gravel	0.029	Clean	0.022
Earth	0.025	With gravel	0.025
Wood	0.012	With brush	0.030
Clay tile	0.014	With cobbles, stones	0.035
Brickwork	0.015	**Floodplains**	
Asphalt	0.016	Pasture, farmland	0.035
Masonry	0.025	Light brush	0.050
Smooth steel	0.012	Heavy brush	0.075
Corrugated metal	0.022	Trees	0.150

Also see NEH654.0609(c) for additional discussion and examples of determining Manning's resistance coefficient.

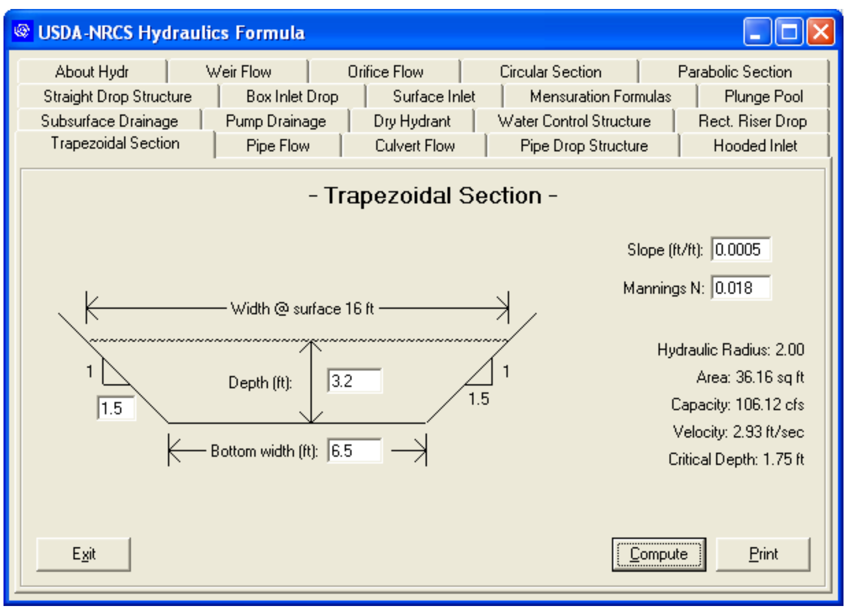
0321.4 Calculations in Uniform Flow

Calculation of discharge using Manning's formula is straightforward for a variety of cross-sections by using the *USDA-NRCS Hydraulics Formula* program. Three examples are illustrated below. An example of normal depth calculation is also shown.

Example 27 – Trapezoidal channel solution using USDA-NRCS Hydraulics Formula program

A trapezoidal channel of width $b = 6.5\ ft$ and side slope $z = 1.5$ flows at a depth $d = 3.2\ ft$ on a slope $S_o = 0.0005$. Use a Manning's $n = 0.018$, and calculate the discharge.

The solution is presented below using the *USDA-NRCS Hydraulics Formula* program for a trapezoidal cross-section:

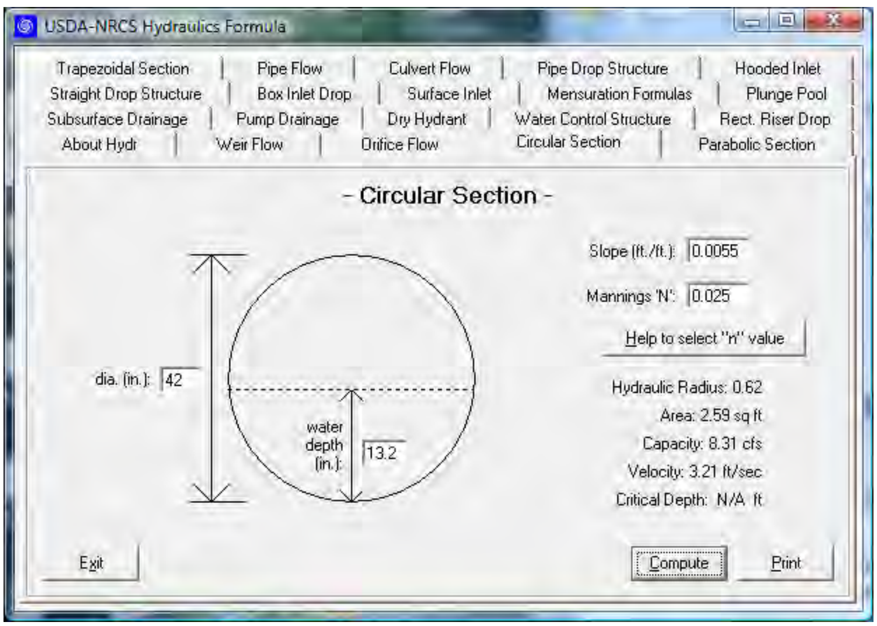


The result is Q = 106.12 cfs, with a velocity V = 2.93 fps. The program also shows the critical depth which is developed in the following section 0322.1.

Note that a trapezoidal cross-section with side slope $z = 0$ represents a rectangular cross section. Also a trapezoidal cross-section with bottom width $b = 0$ represents a triangular cross section.

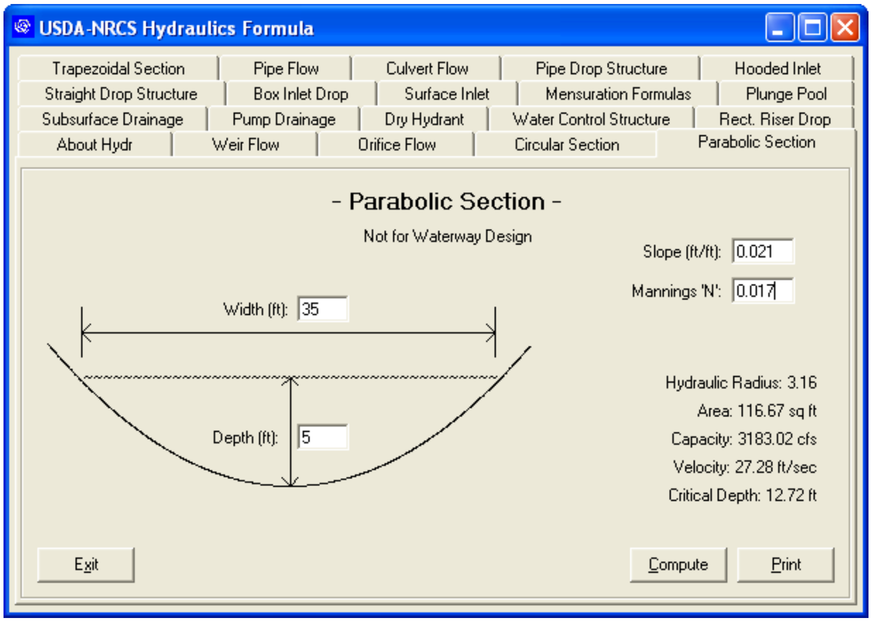
Example 28 – Circular channel solution using USDA-NRCS Hydraulics Formula program
A circular channel of diameter $D = 3.5\ ft = 3.5 \times 12\ in = 42\ in$ flows at a depth $d = 1.1\ ft = 13.2\ in$ on a slope $S_o = 0.0055$. Using a Manning's $n = 0.025$, calculate the discharge.

The solution is presented below using the *USDA-NRCS Hydraulics Formula* program for a circular cross-section:



Example 29 – Parabolic channel solution using USDA-NRCS Hydraulics Formula program
A parabolic channel flows at a depth $d = 5\ ft$ and top width $T = 35\ ft$ on a slope $S_o = 0.021$. Using a Manning's $n = 0.017$, calculate the discharge.

The solution is presented below using the *USDA-NRCS Hydraulics Formula* program for a parabolic cross-section:



Example 30 – Normal depth in a wide channel

Consider a channel with a Manning's $n = 0.015$ laid on a slope $S_o = 0.00461$. Suppose that the channel has a width $b = 30\ ft$ and it carries a flow $Q = 100\ cfs$. Determine the normal depth, d_o.

A wide channel is an approximately rectangular channel whose width b is at least 10 times larger than its depth, i.e., $b/d>10$. For wide channels, the hydraulic radius can be approximated by the flow depth (equation 61). The Manning's equation for this case can be written as:

$$Q = bd_o \frac{C_u}{n} d_0^{2/3} \sqrt{S_o}$$

Solving for the normal depth:

$$d_0 = \left(\frac{Qn}{C_u b \sqrt{S_o}} \right)^{3/5} = \left(\frac{100 \cdot 0.015}{1.486\,(30)\sqrt{0.00461}} \right)^{3/5} = 0.66\ ft$$

0322 Specific Energy in Open Channels

The specific energy in an open channel is the sum of energy heads referred to the channel bed, i.e., the flow depth added to the velocity head:

$$E = d + \frac{V^2}{2g}$$ **[Eq. 76]**

In terms of the discharge *Q*, with *V* = *Q/A*, and *A* = cross-sectional area, the specific energy is written as:

$$E = d + \frac{Q^2}{2gA^2}$$ **[Eq. 77]**

A *specific energy diagram* is a plot of the depth of flow, *d*, versus the specific energy, *E*. The following figure shows the specific energy diagram corresponding to a symmetric trapezoidal open channel of bottom *b = 2 ft* and side slopes *z = 1.5* carrying a flow *Q = 20 cfs*.

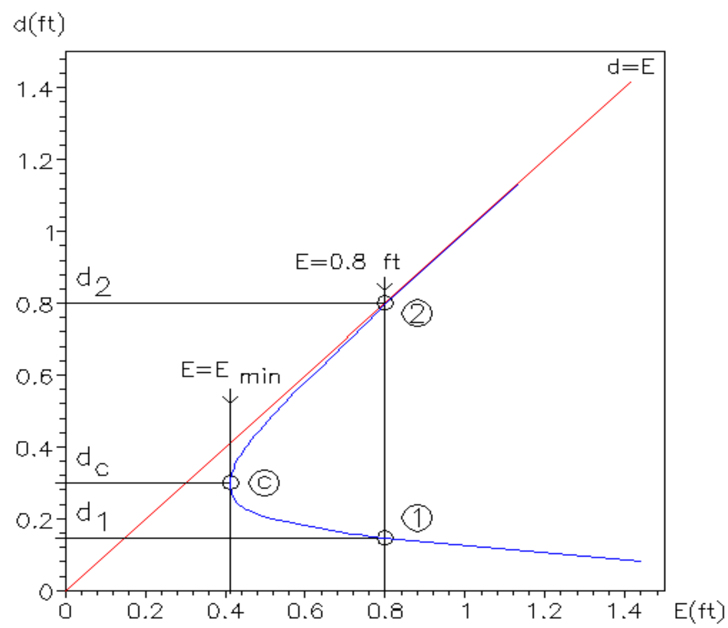


Figure 32. Specific energy curve for a trapezoidal open channel.

In Figure 32, the vertical axis represents the depth of flow and the horizontal axis the specific energy. Notice that the curve approaches the line *d = E* asymptotically as the depth of flow increases. Also, the lower branch of the curve approaches the value *d = 0* as the specific energy *E* increases. The shape of the curve shown is typical of specific energy diagrams in open channel flow. A vertical line corresponding to a specific energy *E = 0.8 ft* is shown. Notice that this line intersects the specific energy curve at two points (1) and (2), indicating that there are two possible flow depths that would produce the same specific energy. These are referred to as *alternate depths*, d_1 and d_2. The specific energy diagram of Figure 32 also shows that there is a point, (c), where the specific energy is a minimum, $E = E_{min}$, for a given cross-section and discharge. This condition is known as *critical flow* and the corresponding flow depth is referred to as *critical depth*. The subject of critical flow is discussed in the following section 0322.1.

Example 31 – Specific energy diagram for a rectangular channel cross-section

A diagram such as that of Figure 32 or 33 can be used to determine, graphically, the alternate depths of flow in a given channel. For a rectangular channel of width $b = 5\ ft$ and carrying a flow of 35 *cfs*, a table of values of the specific energy may be produced:

d(ft)	*A(ft²)*	*V(fps)*	*V²/2g (ft)*	*E(ft)*
0.50	2.50	14.00	3.04	3.54
0.75	3.75	9.33	1.35	2.10
1.00	5.00	7.00	0.76	1.76
1.10	5.50	6.36	0.63	1.73
1.25	6.25	5.60	0.49	1.74
1.50	7.50	4.67	0.34	1.84
1.75	8.75	4.00	0.25	2.00
2.00	10.00	3.50	0.19	2.19
2.25	11.25	3.11	0.15	2.40
2.50	12.50	2.80	0.12	2.62
2.75	13.75	2.55	0.10	2.85
3.00	15.00	2.33	0.08	3.08
3.25	16.25	2.15	0.07	3.32
3.50	17.50	2.00	0.06	3.56
3.75	18.75	1.87	0.05	3.80
4.00	20.00	1.75	0.05	4.05

The resulting specific energy diagram is shown below.

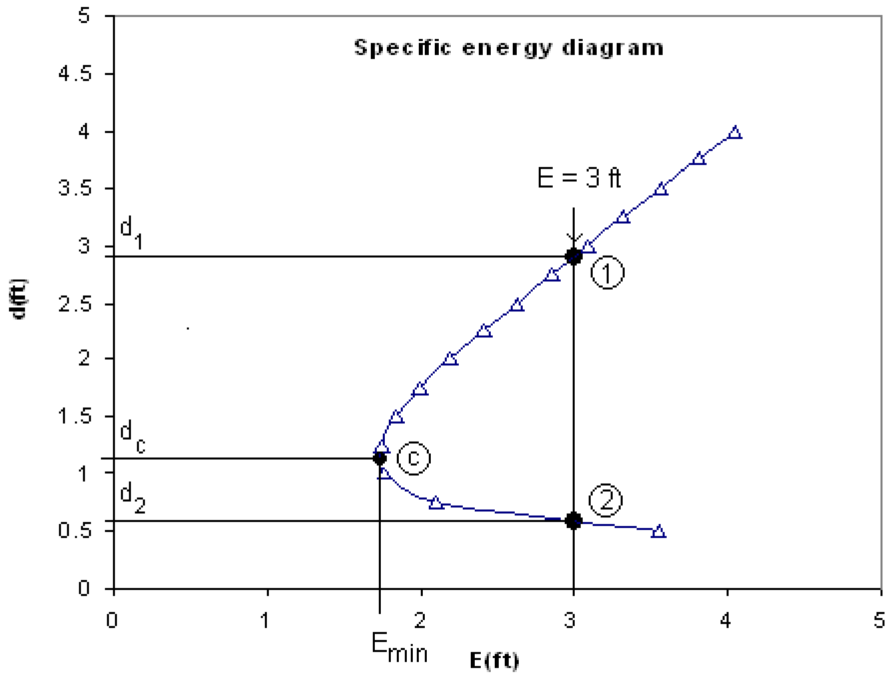


Figure 33. Specific energy curve for a rectangular open channel.

To find the alternate depths corresponding to a specific energy *E = 3 ft* for this case, draw a vertical line at that value of *E* and find the values of *d* where the line *E = 3 ft* intercepts the specific energy diagram. From the figure, the values for the alternate depths are estimated as $d_1 = 2.9$ *ft* and $d_2 = 0.6$ *ft*. The specific energy diagram also reveals that the critical depth is approximately $d_c = 1.1$ *ft* corresponding to a specific energy $E_{min} = 1.7$ *ft*.

0322.1 Critical Flow

The specific energy diagrams shown in Figure 32 and Figure 33 indicate that there is a point (c) where the specific energy is minimal ($E = E_{min}$) for a given cross-section and discharge. This point represents a condition known as *critical flow*, and the corresponding depth of flow is known as the *critical depth*. Critical flow is important in the analysis of open channel flow because it represents conditions of minimal energy. Critical flow can be used for the practical measurement of fluid flows, as in the case of broad-crested weirs or Parshall flumes (see section *0342 – Measurements in Open Channels*).

To determine an equation that describes critical flow conditions, one can start from the definition of the specific energy in terms of the discharge, equation 77, written as:

$$E(d) = d + \frac{Q^2}{2g[A(d)]^2} = d + \frac{Q^2}{2g}\cdot[A(d)]^{-2}$$

Specific energy *E*, as well as, the area *A* of the channel's cross-section, is a function of the flow depth *d*. To find the conditions of minimum energy, take the derivative of *E(d)* with respect to *d* and set it equal to zero:

$$\frac{dE}{d(d)} = 1 + (-2) \cdot \frac{Q^2}{2g} \cdot [A(d)]^{-3} \cdot \frac{dA}{d(d)} = 1 - \frac{Q^2}{gA^3} \cdot \frac{dA}{d(d)} = 0$$

The derivative *dA/d(d)* can be shown to be equal to the top width of the cross-section, as illustrated in the figure below.

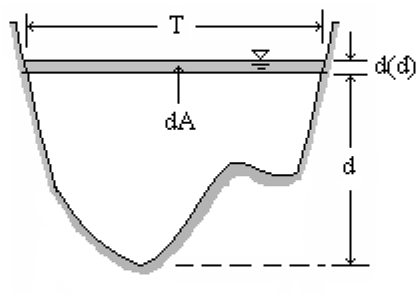


Figure 34. Relationship of cross-sectional area *dA*, flow depth *d(d)*, and top width *T* .

In the figure above, the increment in area, *dA*, due to a small increment in depth *d(d)*, is *dA = T d(d)*; thus, *dA/d(d) = T*, and a critical flow equation can be written as:

$$\frac{Q^2 T_c}{g A_c^3} = 1 \qquad \textbf{[Eq. 78]}$$

Where the subscript, *c* is added to emphasize critical flow conditions.

Re-writing to incorporate the *critical velocity*, $V_c = Q/A_c$:

$$\frac{Q^2 T_c}{g A_c^3} = \frac{(Q/A_c)^2 \cdot T_c}{g \cdot A_c} = \frac{V_c^2}{g \cdot (A_c / T_c)} = 1$$

The ratio *A/T* was defined earlier as the *hydraulic depth* ($D_h = A/T$), thus, an equation for critical velocity can be written as:

$$\frac{V_c^2}{g \cdot (D_h)_c} = 1 \qquad \textbf{[Eq. 79]}$$

The left-hand side of the above equation is the square of the *Froude number*. This dimensionless number is relevant in open channel flow, and is defined, in general, as:

$$Fr = \frac{V}{\sqrt{g \cdot D_h}} \quad \textbf{[Eq. 80]}$$

Thus, the conditions of critical flow require that the Froude number be equal to 1.

Another result that can be derived from the critical conditions equation (equation 79) is that the velocity head is half of the hydraulic depth:

$$\frac{V^2}{2g} = \frac{D_h}{2} \quad \textbf{[Eq. 81]}$$

Critical depth may be calculated with the *USDA-NRCS Hydraulics Formula(s)* program as shown in examples 27 and 29, provided in section 0321.4.

Calculations of critical depth for prismatic open channels often involve solving nonlinear equations, which requires an iterative procedure. The calculations are best done with a spreadsheet, which uses a numerical analysis method to automate the iterative procedure.

0322.1.1 Flow Types

For any energy larger than the minimum, $E > E_{min}$, there are two alternate depths of flow, as indicated in the specific energy diagrams of Figure 32 and Figure 33. One of the depths, d_1 is larger than the critical depth ($d_1 > d_c$), corresponding to a *subcritical flow*, while the second one, d_2, is smaller than the critical depth, and corresponds to a *supercritical flow*.

It can be shown by calculation with the Froude number equation that for *subcritical flow* the Froude number is less than 1 ($Fr<1$), while for *supercritical flow* the Froude number is greater than 1 ($Fr>1$).

If an open channel is laid on a slope S_o such that the normal depth of flow d_o is equal to the critical depth of flow d_c ($d_o = d_c$) for a given discharge Q, then the channel bed slope S_o is said to be the *critical slope* for that flow, i.e., $S_o = S_c$. The critical slope for a channel can be found by replacing $d_o = d_c$ in the Manning's equation. If the channel bed slope is smaller than the critical slope ($S_o < S_c$), the normal depth of flow is larger than the critical depth ($d_o > d_c$), and the channel is said to have a *mild slope*. On the other hand, if the channel bed slope is larger than the critical slope ($S_o > S_c$), the normal depth of flow is smaller than the critical depth ($d_o < d_c$), and the channel is said to have a *steep slope*. The different types of uniform flow possible in an open channel, and their corresponding slopes, are summarized in Table 3 and Figure 35.

Table 3. Types of uniform flow in open channels.

Type of flow	Flow depth	Slope	Type of slope	Froude number

Subcritical	$d_o > d_c$	$S_o < S_c$	*Mild*	*Fr < 1*
Critical	$d_o = d_c$	$S_o = S_c$	*Critical*	*Fr = 1*
Supercritical	$d_o < d_c$	$S_{o.} > S_c$	*Steep*	*Fr > 1*

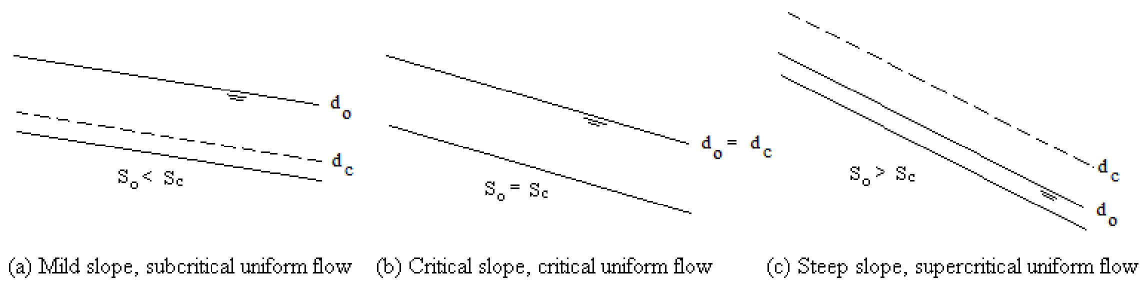


Figure 35. Types of uniform flow in open channels.

It's worth noting that the celerity, or wave speed, *c*, of a surface wave in a shallow open channel of depth *d* is the same as the critical velocity V_c. If the flow is *subcritical*, *c>V*, and wave fronts from a surface disturbance will travel downstream at a speed *c+V >0*, while travelling upstream at a speed *c-V*>0. Here, *V* is the flow velocity. Thus, *surface disturbances in subcritical flow are able to travel both upstream and downstream from their point of origin*. On the other hand, if the flow in *supercritical, V>c*, the velocity of surface disturbances travelling downstream is still positive *c+V >0,* but that of disturbances traveling upstream is negative *c-V<0.* This last result indicates that these disturbances cannot travel upstream. Thus, surface disturbances in supercritical flow can only travel downstream from points of origin.

0322.1.2 Critical Flow in a Rectangular Channel

In a rectangular channel, the area and the top width are *A = bd* and *T=b*. Thus, equation 78 produces the result:

$$\frac{Q^2 T_c}{g A_c^3} = \frac{Q^2 b}{g b^3 d_c^3} = \frac{(Q/b)^2}{g d_c^3} = 1$$

Introducing the *unit discharge* (or *discharge per unit width* of channel):

$$q = \frac{Q}{b} \qquad \textbf{[Eq. 82]}$$

Substituting the *unit discharge* into the equation above, and solving for the critical depth:

$$d_c = \sqrt[3]{\frac{q^2}{g}} \qquad \textbf{[Eq. 83]}$$

Example 32 – Critical flow depth in a rectangular channel

Consider a rectangular channel of width, *b = 30 ft* and carrying a flow *Q = 100 cfs*. Determine the critical depth.

$$q = Q/b = 100\ cfs/30\ ft = 3.33\ ft^2/s,$$

$$d_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{3.33^2}{32.2}} = 0.70\,ft$$

Since for a rectangular channel the hydraulic depth is the same as the flow depth ($D_h = d$), the critical specific energy (minimum energy) is found to be equal to 3/2 of the critical depth (combining equations 76 and 81):

$$E_c = E_{\min} = \frac{3}{2}d_c \qquad \textbf{[Eq. 84]}$$

To find the critical slope, we can use the Manning's equation (equation 73), with $d = d_c$, $A = bd_c$, $P = b + 2d_c$, and $S_o = S_c$:

$$Q = \frac{C_u}{n}\frac{(bd_c)^{5/3}}{(b+2d_c)^{2/3}}S_c^{1/2}$$

Solving for critical slope:

$$S_c = \left(\frac{Qn}{C_u}\right)^2 \frac{(b+2d_c)^{4/3}}{(bd_c)^{10/3}} \qquad \textbf{[Eq. 85]}$$

Example 33 – Critical slope in uniform open-channel flow with rectangular cross-section

A rectangular channel of width *b = 4 ft*, and Manning's *n = 0.012*, is laid on a slope $S_o = 0.00015$ and carries a discharge *Q = 20 cfs*. (a) Determine the critical depth of flow. (b) Determine the critical slope. (c) What type of uniform flow is to be expected in this channel?

(a) First, determining the unit discharge:

$$q = Q/b = (20\ cfs)/(4\ ft) = 5\ ft^2/s$$

The critical depth is calculated as:

$$d_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(5\,ft^2/s)^2}{32.2\,ft/s^2}} = 0.92\,ft$$

(b) The critical slope is:

$$S_c = \left(\frac{Qn}{C_u}\right)^2 \frac{(b+2d_c)^{4/3}}{(bd_c)^{10/3}} = \left(\frac{20\cdot 0.012}{1.486}\right)^2 \frac{(4+2\cdot 0.91)^{4/3}}{(4\cdot 0.91)^{10/3}} = 0.00368$$

(c) Since $S_o < S_c$, the channel slope is *mild*, and a *subcritical* uniform flow is to be expected in this channel.

0322.2 Obstacles in Open Channels

Consider a horizontal rectangular open channel that includes a hump of height Δz at the bottom as illustrated on the left-hand side of Figure 36. The figure also shows the specific energy diagram for a given discharge Q in reference to the original channel bed.

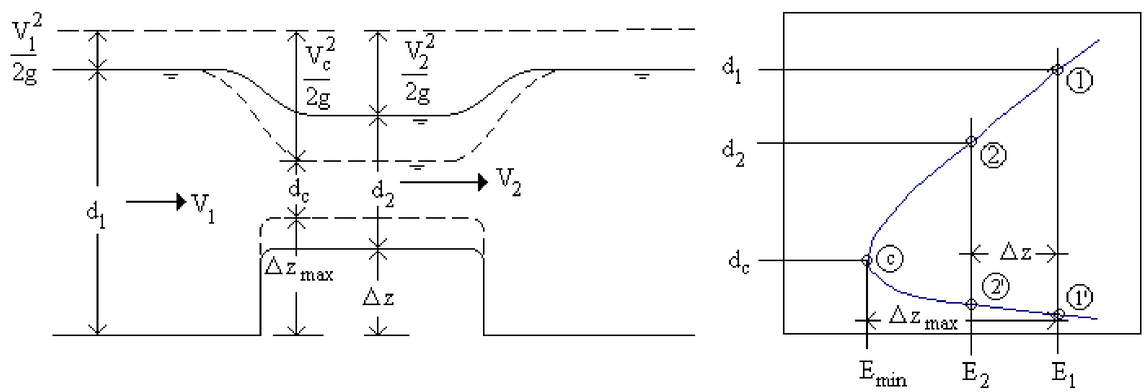


Figure 36. Hump at the bottom of a horizontal rectangular channel.

Figure 36 shows the flow at a subcritical depth d_1 upstream of the hump, and at a subcritical depth d_2 over the hump. The figure also shows the energy heads in sections (1) and (2), assuming no energy losses over the hump. The energy equation written between sections (1) and (2) can be written as:

$$d_1 + \frac{V_1^2}{2g} = \Delta z + d_2 + \frac{V_2^2}{2g} \qquad \textbf{[Eq. 86]}$$

Alternatively, this equation can be written as:

$$E_1 = \Delta z + E_2 \qquad \textbf{[Eq. 87]}$$

The specific energy diagram shows the energy levels $E = E_1$ and $E = E_2$ separated by a distance Δz, as indicated by equation 87, with $E_1 > E_2$. The depth of flow corresponding to energy level $E = E_2$ is $d_2 < d_1$, thus, the water surface over the hump drops as illustrated in Figure 36.

If the flow depth is known at section (1), the specific energy E_1 can be calculated as:

$$E_1 = d_1 + \frac{V_1^2}{2g} = d_1 + \frac{Q^2}{2gb^2 d_1^2}$$

Combining equations 86 and 87 and making use of the continuity equation gives an equation to determine the depth, d_2:

$$d_2 + \frac{Q^2}{2gb^2 d_2^2} = E_1 - \Delta z \quad \textbf{[Eq. 88]}$$

Example 34 – Change in channel bed elevation in a rectangular channel

Refer to Figure 36. A hump of height $\Delta z = 0.25\ ft$ is placed on a rectangular open channel of width $b = 5\ ft$ carrying a discharge $Q = 20\ cfs$. If the flow depth upstream of the hump is $d_1 = 2\ ft$, determine the flow depth atop the hump, d_2.

Applying equation 88 and using several trials, two values were found for d_2, 1.73 ft and 0.42 ft. The iterative calculations may be facilitated by use of a spreadsheet. Since the flow for $d_1 = 2\ ft$ is subcritical, the correct value is d_2 = 1.73 ft (subcritical flow)

To confirm d_2 = 1.73 ft, consider the specific energy diagram in Figure 36. If the flow conditions upstream of the hump correspond to point (1) in the diagram, the flow conditions atop the hump would correspond to point (2). If the flow conditions upstream of the hump correspond to point (1') in the diagram, the flow over the hump would correspond to point (2'). In summary, if the flow upstream of the hump is subcritical, the flow above the hump should be subcritical (or at most, critical), while if the flow upstream of the hump is supercritical, the flow above the hump should be supercritical (or at most, critical).

To check whether the flow at sections (1) and (2) are subcritical or supercritical, one can calculate the Froude number for those sections:

$$Fr_1 = \frac{V_1}{\sqrt{g \cdot d_1}}, Fr_2 = \frac{V_2}{\sqrt{g \cdot d_2}}.$$

The Froude numbers at sections (1) and (2) must both be either smaller than one ($Fr_1 < 1$ and $Fr_2 < 1$) for sub-critical flow, or larger than one ($Fr_1 > 1$ and $Fr_2 > 1$) for supercritical flow.

Example 35 – Calculation of the Froude number in a rectangular channel

For the previous example 34, $d_1 = 2.00\ ft$, $Q = 20\ cfs$, $b = 5\ ft$, *and* $V_1 = Q/A = 2\ ft/s$. The Froude number at section (1) is:

$$Fr_1 = \frac{V_1}{\sqrt{g \cdot d_1}} = \frac{2.00\, ft/s}{\sqrt{32.2\, ft/s^2 \times 2.00\, ft}} = 0.249 < 1,$$

Thus, the flow upstream of the hump for this example is subcritical. Two possible depths of flow atop the hump were found, $d_2 = 1.73\ ft$ and $d_2 = 0.42\ ft$. The Froude numbers corresponding to these depths are:

- For $d_2 = 1.73\ ft$, $Fr_2 = \frac{V_2}{\sqrt{g \cdot d_2}} = \frac{Q/(bd_2)}{\sqrt{g \cdot d_2}} = \frac{20cfs/(5\,ft \times 1.73\,ft)}{\sqrt{32.2\,ft/s^2 \times 1.73\,ft}} = 0.309 < 1$

- For $d_2 = 0.42\ ft$, $Fr_2 = \frac{V_2}{\sqrt{g \cdot d_2}} = \frac{Q/(bd_2)}{\sqrt{g \cdot d_2}} = \frac{20cfs/(5\,ft \times 0.42\,ft)}{\sqrt{32.2\,ft/s^2 \times 0.42\,ft}} = 2.589 > 1$

Thus, $d_2 = 1.73\ ft$ corresponds to a subcritical flow atop the hump, while $d_2 = 0.42\ ft$, corresponds to supercritical flow atop the hump. Since the flow upstream was subcritical, the correct answer is $d_2 = 1.73\ ft$.

There is a possibility that the hump height Δz takes the value Δz_{max} so that the flow conditions on top of the hump are critical. In the specific energy diagram of Figure 36, critical flow conditions correspond to point (c), by making $\Delta z = \Delta z_{max}$, $E_2 = E_{min}$. If the flow atop the hump is critical, the depth of flow d_2 (above the hump) is the critical depth d_c, which, for a rectangular channel, can be calculated using equations 83 or 84.

If critical flow conditions are achieved atop the hump, it may be possible to measure the depth of flow in that location ($d_2 = d_c$). The discharge can then be calculated from equations 82 and 83, combined and rearranged:

$$Q = b\sqrt{g d_c^3} \qquad \textbf{[Eq. 89]}$$

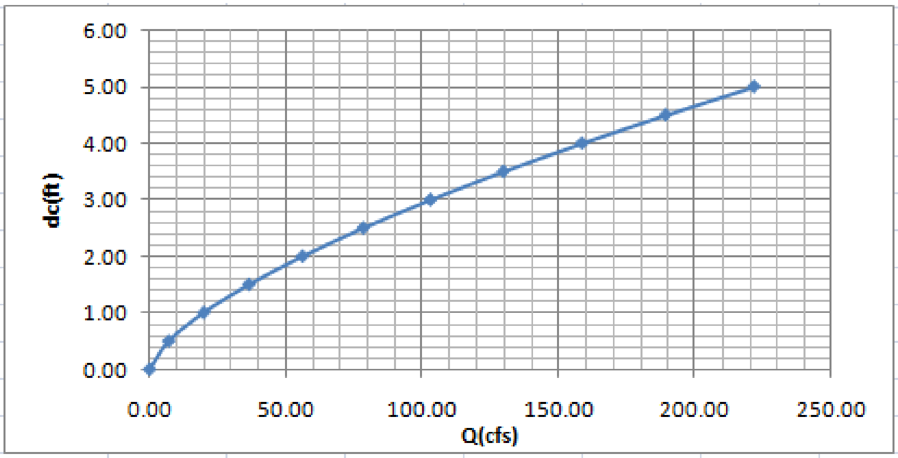
This unique relationship between the flow discharge Q and the depth atop the hump under critical conditions d_c allows the use of relatively high humps as a discharge-measuring device known as a *broad-crested weir*. See section 0342.4, Broad-crested Weirs and and section 0342.6.1, Long-throated Flumes for additional examples and applications.

Example 36 – Rating curve for a broad-crested weir

Draw the rating curve (i.e., Q –vs.-d_c) for a broad-crested weir in a channel that is *3.5-ft* wide, for the range $0 < d_c < 5.0\ ft$.

The table below was developed by applying equation 89 to produce the rating curve as shown below.

d_c (ft)	Q(cfs)
0.00	0.00
0.50	7.02
1.00	19.86
1.50	36.49
2.00	56.17
2.50	78.51
3.00	103.20
3.50	130.05
4.00	158.89
4.50	189.59
5.00	222.05



0323 Momentum Analysis in Open Channels

The energy loss represented by h_f in the energy equation is not easily determined in the analysis of a hydraulic jump (see the following section 0323.1 for hydraulic jump analyses). The principle of impulse-momentum or conservation of momentum is more easily applied (than the energy equation) to analyses involving high internal energy changes, such as hydraulic jump or sluice gate flow analyses. Momentum may be thought of as "mass-in-motion" and an impulse as change in momentum. The principle of conservation of momentum is used to determine forces on moving fluids. Consider, for example, the case of the flow under a sluice gate illustrated in Figure 26. Figure 37 below shows a sluice-gate control volume with the forces acting on it, and the flow of momentum through the control surfaces.

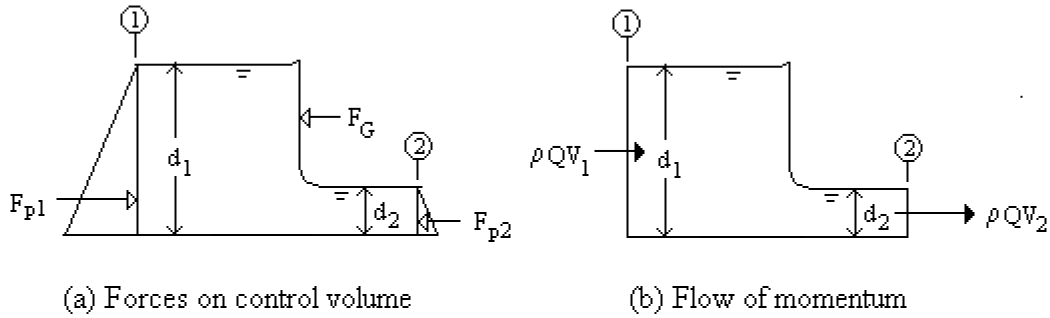


Figure 37. Forces and flow of momentum for sluice gate flow.

Figure 37(a) shows hydrostatic forces F_{p1} and F_{p2} acting in the upstream and downstream sections of the control volume for the flow under a sluice gate. Included also in Figure 37(a) is the force F_G that the gate exerts on the control volume. By Newton's third law (principle of action-reaction), the flow exerts a force $-F_G$ in the opposite direction on the gate. Figure 37(b) shows the flow of momentum in (ρQV_1) and out (ρQV_2) of the control volume through the control surfaces at sections (1) and (2), respectively.

The principle of impulse-momentum states that the sum of forces on the control volume is equal to the net flow of momentum out of the control volume, i.e., momentum flow out minus momentum flow in. This principle can be expressed as the following vector equation:

$$\Sigma F = \Delta(\rho QV) = (\rho QV)_{out} - (\rho QV)_{in} \quad \textbf{[Eq. 90]}$$

Specifically, for the case illustrated in Figure 37:

$$F_{p1} - F_G - F_{p2} = \rho QV_2 - \rho QV_1$$

from which it follows that:

$$F_G = (F_{p1} + \rho QV_1) - (F_{p2} + \rho QV_2)$$

The hydrostatic forces F_{p1} and F_{p2} can be calculated by using equation 16:

$$F_{p1} = \omega(h_cA)_1, \quad F_{p2} = \omega(h_cA)_2$$

Where h_cA stands for the first moment of area with respect to the free surface of a given cross-section (h_c is the depth of the centroid of the cross-section, and A is the area). Also, replacing $V_1 = Q/A_1$ and $V_2 = Q/A_2$, and using $\rho = \omega/g$, the expression for F_G becomes:

$$F_G = \omega\left[\left(h_cA + \frac{Q^2}{gA}\right)_1 - \left(h_cA + \frac{Q^2}{gA}\right)_2\right]$$

The quantities between parentheses in the equation above is defined as the *momentum function* for open channel flow:

$$M_F = h_c A + \frac{Q^2}{gA} \qquad \textbf{[Eq. 91]}$$

The force on the gate can now be written as:

$$F_G = \omega\left[(M_F)_1 - (M_F)_2\right] \qquad \textbf{[Eq. 92]}$$

For a rectangular channel, $A = bd$, and $h_c = d/2$, thus, $h_cA = ½\ bd^2$. Also, using the concept of the unit discharge (discharge per unit width), $q = Q/b$, the term $Q^2/gA = q^2b/gd$. The momentum function (equation 91) becomes:

$$M_F = h_c A + \frac{Q^2}{gA} = \frac{1}{2}bd^2 + \frac{q^2 b}{gd} = b\left(\frac{d^2}{2} + \frac{q^2}{gd}\right)$$

Continuing, a *unit momentum function* (or *momentum function per unit width*) for a rectangular open channel can be defined as:

$$m_F = \frac{M_F}{b} = \frac{d^2}{2} + \frac{q^2}{gd} \qquad \textbf{[Eq. 93]}$$

A *unit momentum function diagram* for a rectangular cross-section is a plot of the channel depth (d) against the unit momentum function (m_F), as illustrated in the following example.

Example 37 – Momentum function diagram for a rectangular channel

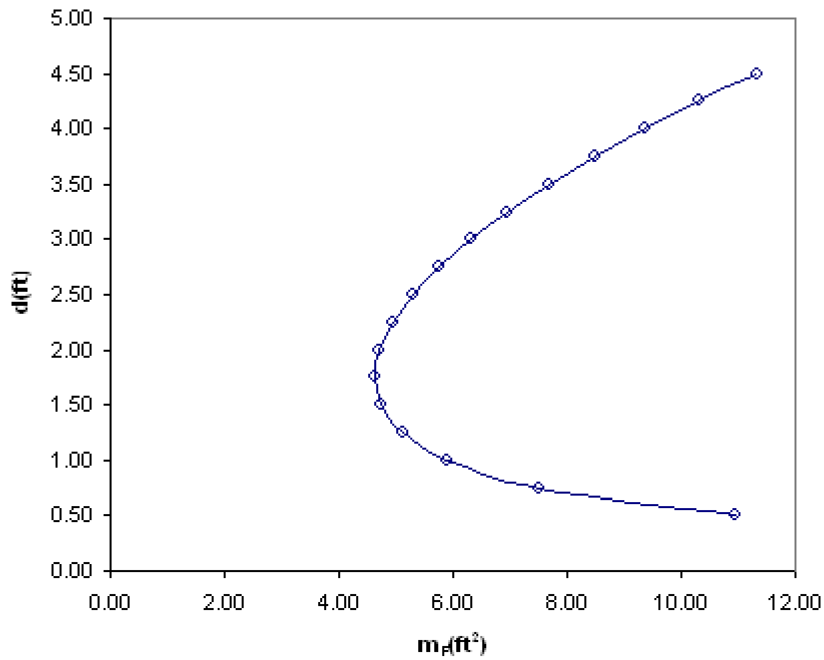
A rectangular channel of width $b = 10\ ft$ carries a discharge $Q = 132\ ft^3/s$; produce a unit momentum function diagram for this channel.

The unit discharge is:

$$q = Q/b = (132\ ft/s^3)/(10\ ft) = 13.2\ ft^2/s.$$

The unit momentum function diagram, below, was developed by applying equation 93:

d(ft)	$m_F(ft^2)$
0.50	10.95
0.75	7.50
1.00	5.91
1.25	5.11
1.50	4.73
1.75	4.62
2.00	4.71
2.25	4.94
2.50	5.29
2.75	5.75
3.00	6.30
3.25	6.95
3.50	7.67
3.75	8.47
4.00	9.35
4.25	10.30
4.50	11.33



Example 38 – Calculation of force on sluice gate

In example 15 (the sluice gate of Figure 26), the values $b = 10\ ft$, $d_1 = 3.5\ ft$, and $d_2 = 1.0\ ft$ were used to calculate $Q = 132\ ft/s^3$, i.e, $q = Q/b = 13.2\ ft^2/s$. Calculate the force that the flowing water exerts on the gate.

The force can be calculated by using the unit momentum function as follows:

$$(m_F)_1 = \frac{d_1^2}{2} + \frac{q^2}{gd_1} = \frac{(3.5\,ft)^2}{2} + \frac{(13.2\,ft^2/s)^2}{32.2\,ft/s^2 \times 3.5\,ft} = 7.67\,ft^2$$

$$(m_F)_2 = \frac{d_2^2}{2} + \frac{q^2}{gd_2} = \frac{(1.0\,ft)^2}{2} + \frac{(13.2\,ft^2/s)^2}{32.2\,ft/s^2 \times 1.0\,ft} = 5.91\,ft^2$$

With $\omega = 62.4\ lb/ft^3$, the force on the gate, as given by equation 92, is:

$$F_G = \omega\left[(M_F)_1 - (M_F)_2\right] = \omega \cdot b \cdot \left[(m_F)_1 - (m_F)_2\right]$$

$$F_G = 62.4lb/\,ft^3 \cdot 10\,ft \cdot \left[7.67\,ft^2 - 5.91\,ft^2\right] = 1098.24lb$$

An analysis of the unit momentum function diagram used in example 37 above indicates that there are two possible depths associated with a given value of m_F. This is illustrated in the figure below. These two depths are referred to as *conjugate depths*.

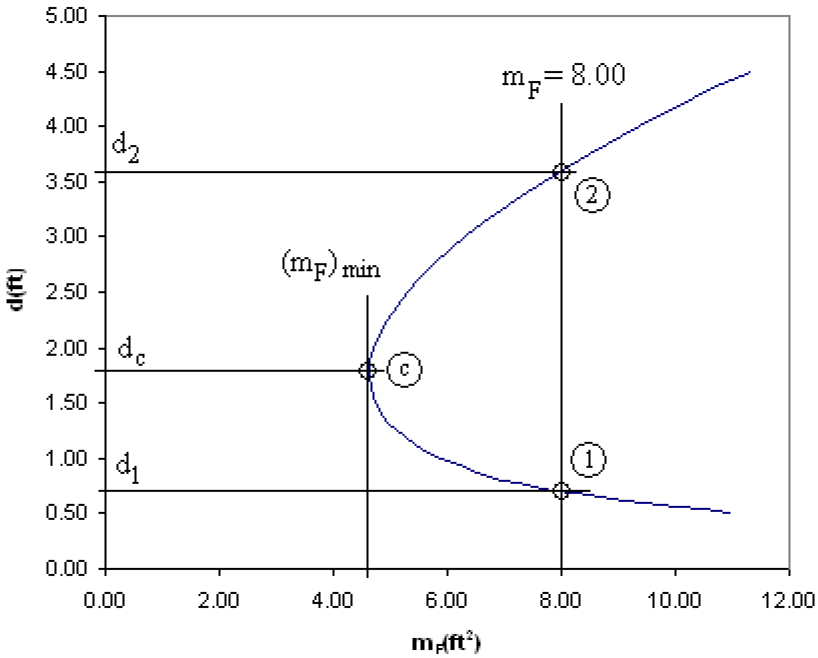


Figure 38. Conjugate depths in the unit momentum function diagram for a rectangular open channel.

The unit momentum function diagram also shows that there is a point where the unit momentum function becomes a minimum, $m_F = (m_F)_{min}$. It can be shown that this point of minimum momentum function corresponds to critical flow. Thus, for a given value of the momentum function there is a subcritical and a supercritical depth of flow, the conjugate depths.

0323.1 Hydraulic Jumps

A *hydraulic jump* consists of a sudden increase of water depth in an open channel from a supercritical depth ($d_1 < d_c$) to a subcritical depth ($d_2 > d_c$). A hydraulic jump could occur at the foot of a spillway as illustrated in the following figure.

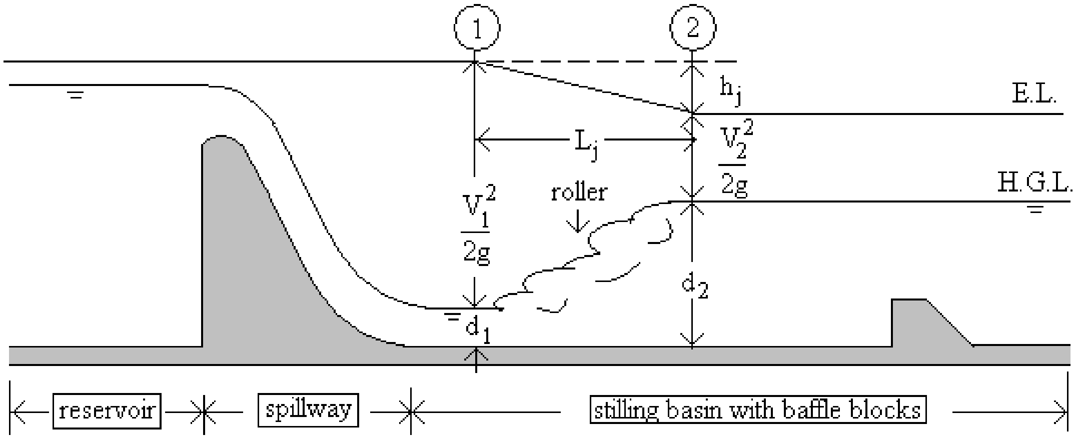


Figure 39. Hydraulic jump produced by a stilling basin at the base of a spillway.

The figure below, shows a photograph of a hydraulic jump at the base of a model of a dam spillway.

Figure 40. Hydraulic jump observed at the foot of a model spillway (Courtesy of the Utah Water Research Laboratory).

The hydraulic jump is typically an abrupt raise in the water surface showing a rough surface with strong turbulence and producing a large amount of air entrainment. This area of strong turbulence and marked air entrainment is referred to as the *roller* of the hydraulic jump. The sketch of the hydraulic jump shown in Figure 39 indicates an energy loss h_j through the jump.

Since energy is not conserved through the jump, analysis of the hydraulic jump is better performed through the use of the impulse-momentum principle. Forces on a control volume enclosing the jump, as well as the momentum flow through the corresponding control surfaces, are shown below.

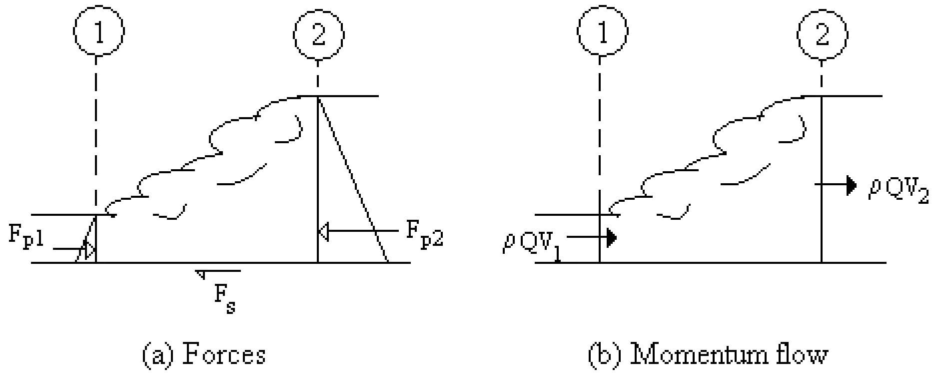


Figure 41. Forces and flow of momentum for a hydraulic jump.

Besides the hydrostatic forces F_{p1} and F_{p2}, the control volume is affected by a shear force at the wetted perimeter F_s. Using the principle of impulse-momentum for the jump results in an equation similar to that of the force on a sluice gate (equation 92):

$$F_S = \omega\left[(M_F)_1 - (M_F)_2\right]$$

Since the hydraulic jump occurs along a relatively small length of the channel, the shear force F_s is negligible, and the impulse-momentum principle for the hydraulic jump results in:

$$(M_F)_1 = (M_F)_2$$

If the hydraulic jump occurs in a rectangular channel, the principle of impulse-momentum produces the result:

$$\frac{d_1^2}{2} + \frac{q^2}{gd_1} = \frac{d_2^2}{2} + \frac{q^2}{gd_2}$$

The depths of flow upstream (d_1) and downstream (d_2) of the jump are known as *conjugate depths*.

Algebraic manipulation of the above equation gives the following results for a horizontal rectangular-channel hydraulic jump:

- Unit discharge (discharge per unit width):

$$q = \sqrt{\frac{g\, d_1 d_2 (d_1 + d_2)}{2}} \qquad \textbf{[Eq. 94]}$$

- Ratio of depths:

$$\frac{d_2}{d_1} = \frac{1}{2} \cdot \left[-1 + \sqrt{1 + \frac{8q^2}{gd_1^3}} \right] \quad \textbf{[Eq. 95]}$$

or,

$$\frac{d_1}{d_2} = \frac{1}{2} \cdot \left[-1 + \sqrt{1 + \frac{8q^2}{gd_2^3}} \right] \quad \textbf{[Eq. 96]}$$

The *energy head loss* is equal to the difference in specific energy before and after the jump:

$$h_j = \frac{(d_2 - d_1)^3}{4d_1 d_2} \quad \textbf{[Eq. 97]}$$

The length of the jump, L_j, cannot be determined from energy or momentum considerations. However, experimental results reveal that:

$$4 < \frac{L_j}{d_2} < 6 \quad \textbf{[Eq. 98]}$$

Thus, the average value can be used as a first approximation to the jump length:

$$L_j = 5d_2 \quad \textbf{[Eq. 99]}$$

Example 39 – Discharge, head loss, and length of a hydraulic jump in a rectangular channel

A hydraulic jump is observed in a rectangular channel and the upstream and downstream depths are measured to be $d_1 = 0.50\ ft$ and $d_2 = 3.5\ ft$, respectively. Please refer to Figure 39 as a schematic. Determine (a) the discharge per unit width, (b) the energy head loss through the jump, and (d) an approximation to the jump length.

The unit discharge is (equation 94):

$$q = \sqrt{\frac{g\, d_1 d_2 (d_1 + d_2)}{2}} = \sqrt{\frac{32.2\, ft/s^2 \times 0.50\, ft \times 3.5\, ft \times (0.5\, ft + 3.5\, ft)}{2}} = 10.62\, ft^2/s$$

The energy head loss is (equation 97):

$$h_j = \frac{(d_2 - d_1)^3}{4d_1 d_2} = \frac{(3.5\, ft - 0.5\, ft)^3}{4 \times 0.50\, ft \times 3.5\, ft} = 3.86\, ft$$

An approximation to the jump length is (equation 99):

$$L_j = 5d_2 = 5\times 3.5\ ft = 17.5\ ft$$

Example 40 – Flow depth, head loss, and length of hydraulic jump in a rectangular channel

A hydraulic jump takes place in a rectangular channel of width $b = 5.5\ ft$ that carries a discharge $Q = 25\ cfs$. Please refer to Figure 39 as a schematic. If the depth upstream of the jump is determined to be $d_1 = 0.75\ ft$, by means of water surface profile calculations, (see section 0324) determine: (a) the depth downstream of the jump, (b) the energy head loss through the jump, and (c) an approximation to the jump length.

The unit discharge is:

$$q = Q/b = 25\ cfs/5.5\ ft = 4.55\ ft^2/s.$$

From equation 95, d_2 is found:

$$d_2 = \frac{d_1}{2}\cdot\left[-1+\sqrt{1+\frac{8q^2}{gd_1^3}}\right] = \frac{0.75\,ft}{2}\cdot\left(-1+\sqrt{1+\frac{8\times(4.55\,ft^2/s)^2}{32.2\,ft/s^2\times(0.75\,ft)^3}}\right) = 0.987\ ft$$

The energy head loss is (equation 97):

$$h_j = \frac{(d_2-d_1)^3}{4d_1d_2} = \frac{(0.987\,ft-0.75\,ft)^3}{4\times 0.987\,ft\times 0.75\,ft} = 0.0045\,ft$$

An approximation to the jump length is (equation 99):

$$L_j = 5d_2 = 5\times 0.987\ ft = 4.94\ ft$$

Figure 42 below illustrates a hydraulic jump occurring over an obstacle or a baffle block.

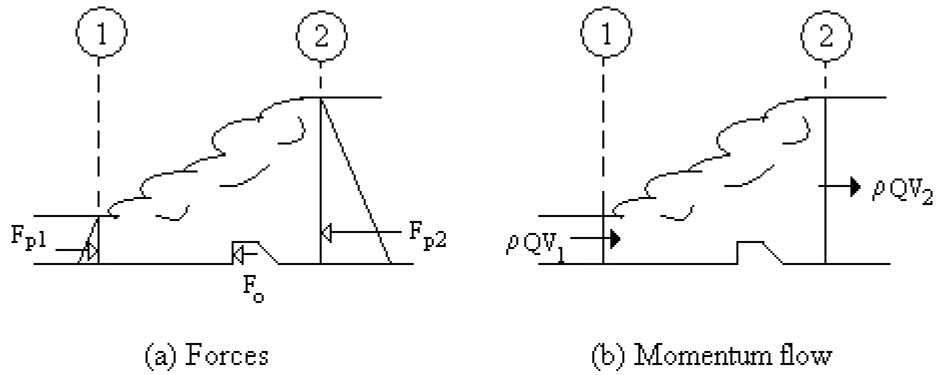


Figure 42. Hydraulic jump over a baffle block.

For the case illustrated in Figure 42, equations 94 through 97 no longer apply because the force that the obstacle or block exerts on the flow is not negligible. However, the principle of impulse-momentum may be applied in the same manner as it was for sluice-gate flow:

$$F_o = \omega\left[(M_F)_1 - (M_F)_2\right] \qquad \textbf{[Eq. 100]}$$

Where $(M_F)_1$ and $(M_F)_2$ are the momentum functions (equation 92) for the sections upstream and downstream of the hydraulic jump, respectively. For a rectangular cross-section, the above equation can be written as:

$$F_o = \omega \cdot b \cdot [(m_F)_1 - (m_F)_2] \qquad \textbf{[Eq. 101]}$$

Where ω is the specific weight of water, b is the channel width, and the unit momentum function, m_F, is calculated by equation 93.

Example 41 – Calculation of force on an obstacle producing a hydraulic jump in a rectangular channel

A hydraulic jump in a rectangular channel of width $b = 10\ ft$ is produced by an obstacle at the channel bed. Please refer to Figure 42 as a schematic. If the depths upstream and downstream of the jump are determined to be $d_1 = 0.95\ ft$ and $d_2 = 1.25\ ft$, respectively, and the discharge in the channel is $Q = 80\ ft/s^3$. Calculate the force that the flowing water exerts on the obstacle.

The unit discharge is:

$$q = Q/b = 80\ cfs/10\ ft = 8\ ft^2/s$$

The force can be calculated by using the unit momentum function (equation 93):

$$(m_F)_1 = \frac{d_1^2}{2} + \frac{q^2}{gd_1} = \frac{(0.95\,ft)^2}{2} + \frac{(8\,ft^2/s)^2}{32.2\,ft/s^2 \times 0.95\,ft} = 2.543\,ft^2$$

$$(m_F)_2 = \frac{d_2^2}{2} + \frac{q^2}{gd_2} = \frac{(1.25\,ft)^2}{2} + \frac{(8\,ft^2/s)^2}{32.2\,ft/s^2 \times 1.25\,ft} = 2.37\,ft^2$$

With $\omega = 62.4\ lb/ft^3$, the force on the obstacle, as given by equation 100, is:

$$F_0 = \omega[(M_F)_1 - (M_F)_2] = \omega \cdot b \cdot [(m_F)_1 - (m_F)_2]$$

$$F_0 = 62.4 lb/ft^3 \cdot 10\,ft \cdot [2.543\,ft^2 - 2.371\,ft^2] = 107.6\,lb$$

0324 Varying Open Channel Flow

Uniform flow has a contant depth and is achieved when the driving force (gravity) is in balance with the resisting force (shear forces) on the channel boundary. The entrance from a reservoir into a long open channel, as illustrated in Figure 43, below, may include a zone of varying flow depth, before uniform flow is achieved.

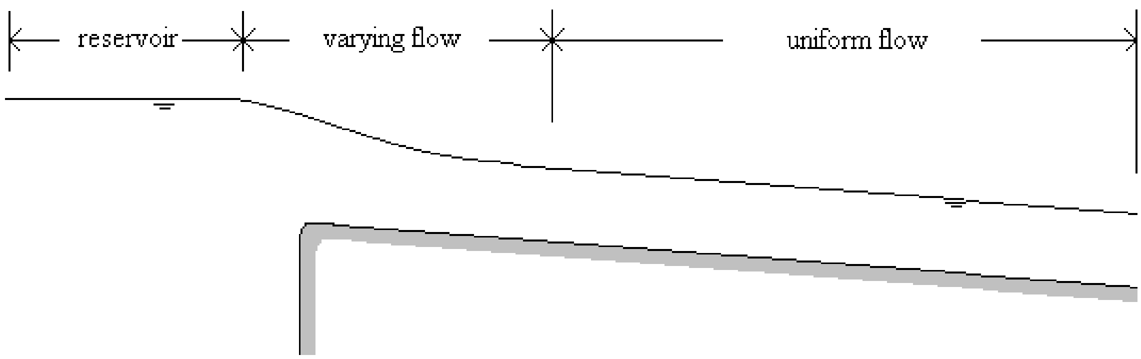


Figure 43. Varying flow from a reservoir leading to uniform flow in an open channel.

A zone of varying flow depth, as the one illustrated in Figure 43, is referred to as *gradually varied flow* (GVF). Gradually varied flow zones may be very long. Hydraulic jumps (Figure 39) or flows over an obstacle (Figure 42), on the other hand, are examples of *rapidly varied flow*, with the flow depth changing quickly. Flow depths generally vary over the entire length of natural channels.

0324.1 Gradually-varied Flow

Figure 25 shows the energy heads in a gradually varied flow between two sections separated by a horizontal distance Δx. The slopes shown in that figure include:

- Slope of the energy line, $S_f = h_f / L$, where h_f = energy head loss, and L = length between sections (1) and (2) measured along channel.
- Slope of the water surface, $S_w = (WS_1 - WS_2)/\Delta x$, where WS_1 and WS_2 are the water surface elevations at sections (1) and (2)
- Slope of the channel bed, $S_o = (z_1 - z_2)/\Delta x$, where z_1 and z_2 are the bed elevations at sections (1) and (2)

Because typically the channel bed slope S_o is small, the length of channel L is approximately equal to the horizontal length Δx between sections (1) and (2), i.e., $L \approx \Delta x$, and $S_f \approx h_f / \Delta x$.

Two equations useful in calculating GVF parameters include the energy equation (see section 0317.4), written as:

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f \qquad \textbf{[Eq. 102]}$$

And Manning's equation, used to estimate the energy slope, S_f:

$$S_f = \left(\frac{Q \cdot n}{C_u}\right)^2 \cdot \frac{1}{(A \cdot R_h^{2/3})^2} \qquad \textbf{[Eq. 103]}$$

Figures 44 and 45 illustrate two common forms of GVF. In Figure 44 a uniform flow in a mild-slope channel approaches an overfall. The GVF curve approaches the line of uniform flow (d_o) asymptotically at (a), while approaching the line of critical flow (d_c) almost perpendicularly at (b). The GVF curve in the figure below is referred to as a *drawdown curve*.

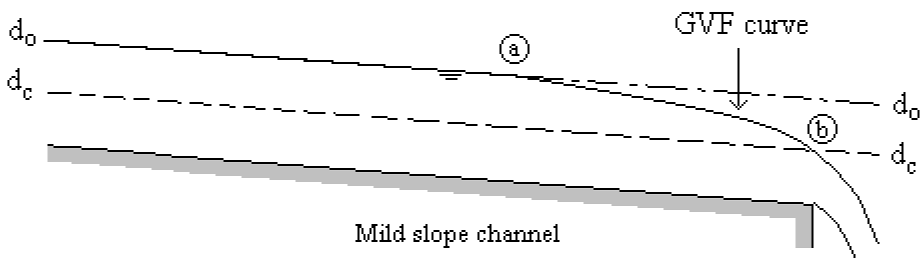


Figure 44. Gradually-varied flow (GVF) near an overfall.

Another example of GVF is shown in Figure 45. In this case, a weir across a channel forces the water depth above the normal depth of flow. The figure shows a GVF curve that approaches the normal depth line asymptotically at point (a). The curve in the figure below is referred to as a *backwater curve*.

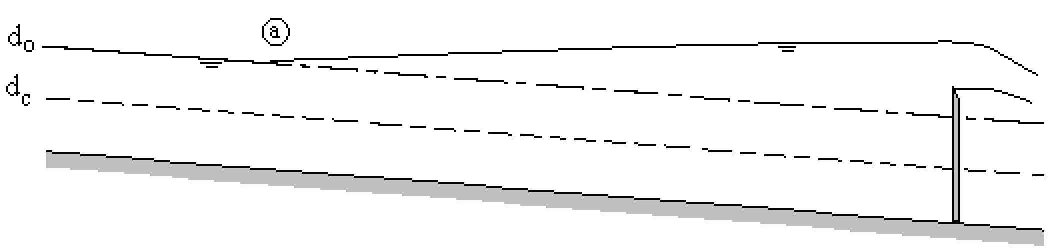


Figure 45. Gradually-varied flow (GVF) produced by a weir.

0324.2 Classification of Gradually-Varied Flow

GVF curves cannot cross the lines of normal depth (d_o) or of critical depth (d_c). Thus, *backwater curves* or *drawdown curves* must be contained within the region limited by the channel bed and the closest of the d_o or d_c lines, the region between those lines, or the region above the highest of the d_o and d_c lines. These two or three regions are indicated in Figure 46. GVF curves are classified according to the type of slope of the channel and the region where they occur. For example, a curve above the d_o line in a mild-slope channel would be classified as *M1*, and so on. The different types of gradually varied flow curves are illustrated in Figure 46.

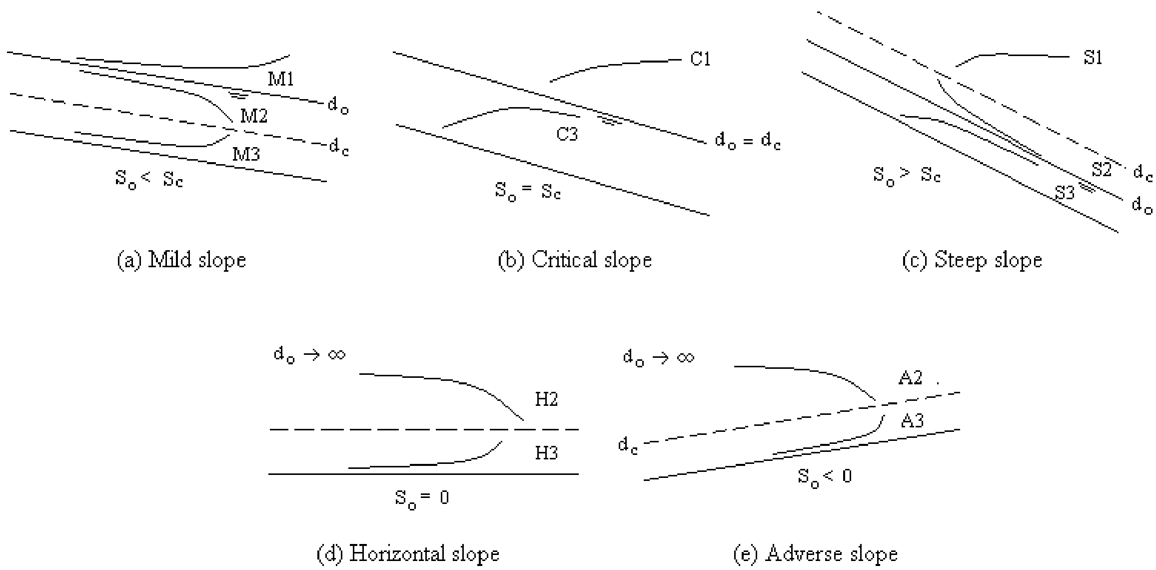


Figure 46. Classification of gradually-varied flow (GVF).

The curves of Figure 46 can be used to sketch the type of gradually-varied flow expected on a given channel.

The following figure illustrates the type of curves that can be generated by a sluice gate placed across the channel, so that it produces a supercritical flow under the gate. The channel ends with an overfall.

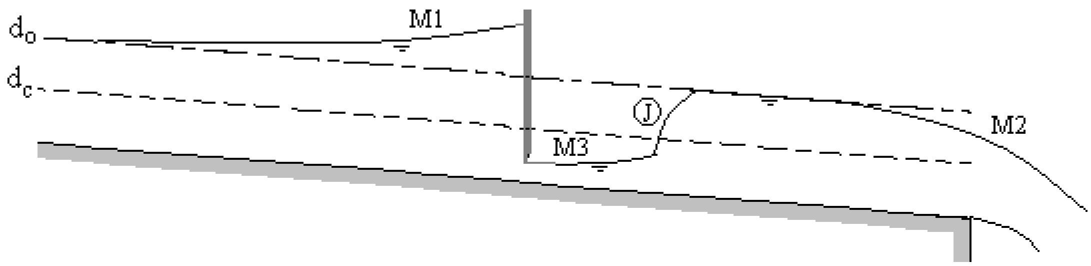


Figure 47. Gradually-varied flow (GVF) curves in a mild-slope open channel with a sluice gate and overfall.

The sluice gate in Figure 47 produces an *M1* curve upstream, and an *M3* curve downstream of the gate. The *M3* curve ends in a hydraulic jump (J) that raises the water level to the normal depth of flow before reaching the overfall. The overfall produces an *M2* curve at the downstream end of the channel.

0324.3 Standard step method

Manning's equation provides a good estimate of flow depth, when uniform flow conditions exist. See section 0321.4, Calculations in Uniform Flow. Because uniform flow conditions of constant depth and discharge do not normally exist in natural streams and in many reaches of constructed channels, other methods, such as the standard step, are needed to more accurately calculate flow profiles.

The *standard step method* calculates a gradually varied flow (GVF) profile by solving the energy equation with an iterative procedure. One way of applying the method is by varying the flow depths. The calculations start at a point where the depth is known and the depth is changed by small increments or decrements until reaching a specified depth value. For each depth increment or decrement, the distance between sections, Δx, for the given depth change, is calculated.

To develop an equation for the standard step method one may start from equation 102, re-written as follows:

$$z_1 + E_1 = z_2 + E_2 + h_f \qquad \textbf{[Eq. 104]}$$

Using z_1-z_2 = $S_o\Delta x$ and h_f = $S_f\Delta x$, we can solve for Δx, the horizontal distance between sections (1) and (2):

$$\Delta x = \frac{E_1 - E_2}{S_f - S_o} \qquad \textbf{[Eq. 105]}$$

The value of the energy slope to use is based on the Manning's equation by using the average velocity and hydraulic radius for sections (1) and (2):

$$\overline{V} = \frac{1}{2}(V_1 + V_2), \quad \overline{R}_h = \frac{1}{2}(R_{h1} + R_{h2}) \qquad \textbf{[Eq. 106]}$$

The energy slope is calculated using Manning's equation as:

$$S_f = \left(\frac{n \cdot \overline{V}}{C_u \cdot \overline{R}_h^{2/3}} \right)^2 \qquad \textbf{[Eq. 107]}$$

The calculation procedure starts by selecting a depth d_1 and then postulating a depth $d_2 = d_1 \pm \Delta d$, e.g., you could have $d_1 = 2.5\ ft$ and $d_2 = 2.6\ ft$ (with $\Delta d = 0.1\ ft$). Then, proceeding to calculate the areas (A_1, A_2), wetted perimeters (P_1, P_2), hydraulic radii (R_{h1}, R_{h2}), velocities (V_1, V_2), specific energies (E_1, E_2), average hydraulic radius (R_h), average velocity (V), energy slope (S_f), and finally the increment Δx from equation 105 is calculated. The procedure is repeated then by taking $d_1 = d_2$ and postulating a new value of d_2. Tables of the calculation results are shown in the following example.

Example 42 – Gradually-varied flow calculation in a rectangular channel

A rectangular flume is *5-ft* wide ($b = 5ft$) and carries a flow $Q = 60\ cfs$. The bed slope is $S_o = 0.0006$, and the Manning's resistance coefficient is $n = 0.012$. At a certain section the depth is $d_1 = 3\ ft$. Find the distance Δx to the section where the depth is *2.5 ft*.

Using depth decrements of $\Delta d = -0.1\ ft$, the depths $d = 3.0\ ft$, *2.9 ft*, *2.8 ft*, *2.7 ft*, *2.6 ft*, *and 2.5 ft* are used in the calculation. Calculations are shown for the depths, *3.0 ft and 2.9 ft*. For a rectangular cross section the area and wetted perimeter are given by $A = bd = 5d$, $P = b + 2d = 5+2d$, respectively. Thus:

- For $d_1 = 3\ ft$, $A_1 = 5\times 3 = 15\ ft$, $P_1 = 5 + 2\times 3 = 11\ ft$, $R_{h1} = A_1/P_1 = 1.36\ ft$, $V_1 = Q/A_1 = 4\ ft/s$, $E_1 = d_1 + V_1^2/2g = 3.2484\ ft$

- For $d_2 = 2.9\ ft$, $A_2 = 5\times 2.9 = 14.5\ ft$, $P_2 = 5 + 2\times 2.9 = 10.8\ ft$, $R_{h2} = A_2/P_2 = 1.34\ ft$, $V_2 = Q/A_2 = 4.14\ ft/s$, $E_2 = d_2 + V_2^2/2g = 3.166\ ft$

The average velocity and hydraulic radius are:

$$\overline{V} = (V_1+V_2)/2 = 4.07\ ft/s$$

$$\overline{R}_h = (R_{h1}+R_{h2})/2 = 1.35\ ft$$

The energy slope is calculated as follows:

$$S_f = \left(\frac{n \cdot \bar{V}}{C_u \cdot \bar{R}_h^{2/3}} \right)^2 = \left(\frac{0.012 \times 4.07}{1.486 \times (1.35)^{2/3}} \right)^2 = 0.000721$$

Finally, the distance between cross sections is calculated as:

$$\Delta x = \frac{E_1 - E_2}{S_f - S_o} = \frac{3.2484\,ft - 3.166\,ft}{0.000721 - 0.0006} = 680\,ft$$

The standard step solution is presented in the following tables.

Depth	Area	Wetted Perimeter	Hydraulic Radius	Flow Velocity	Specific Energy	Average Hyd. R.	Average Velocity
d	***A***	***P***	***R = A/P***	***V***	***E***	***R̄***	***V̄***
3.000	15.000	11.000	1.364	4.000	3.248	-	-
2.900	14.500	10.800	1.343	4.138	3.166	1.353	4.069
2.800	14.000	10.600	1.321	4.286	3.085	1.332	4.212
2.700	13.500	10.400	1.298	4.444	3.007	1.309	4.365
2.600	13.000	10.200	1.275	4.615	2.931	1.286	4.530
2.500	12.500	10.000	1.250	4.800	2.858	1.262	4.708

Depth	Energy Slope	Spec. En. Increment	Distance Increment	Cumulative Distance
d	***S***	***ΔE***	***Δx***	***x***
3	-	-	-	0
2.9	0.000721	0.0826	680.11	680.11
2.8	0.00079	0.0807	425.5	1105.61
2.7	0.000867	0.0785	293.53	1399.14
2.6	0.000957	0.076	213.01	1612.15
2.5	0.001059	0.073	158.91	1771.05

The cumulative distance to the section where the depth is *2.5 ft* is shown to be *1771 ft*.

The best results in the standard step method are achieved by using an increment or decrement of depth as small as possible. Thus, implementation of the standard step solution in spreadsheet software facilitates the calculation, especially for the trapezoidal, parabolic, and circular shapes that occur in prismatic open channels.

To achieve best results in the standard step method, the step computations should be carried upstream for subcritical flows and downstream for supercritical flows.

For analyses of gradually-varied flow in prismatic or natural channels, one can use the U.S. Army Corps of Engineers' Hydrologic Engineering Center's HEC-RAS software.

(RAS stands for *River Analysis System*.) The latest version of HEC-RAS can be downloaded at

http://www.hec.usace.army.mil/software/hec-ras/

HEC-RAS utilizes an iterative standard step method to solve for gradually varied flow. Details on the operation of HEC-RAS are available at the website shown above.

0325 Sediment Transport

Earth-lined channels may carry sediments if the water velocities are large enough to produce erosion of the channel linings. Rivers may carry significant sediment load when the water discharge increases. The ability of a river to erode and carry sediments depends on the hydraulic characteristics of the stream as well as on the sediment properties. Refer to NEH 654, Chapter 13, for additional sediment transport information.

0325.1 Sediment Properties

The size of sediments can be determined by performing a sieve test on a sample of sediments. For very fine sediments a settling test performed in a settling tube may be necessary to determine the size distribution of the particles. Typically, sediment size follows a log-normal distribution of probabilities, with a *median size* D_{50}, meaning the size for which 50% of the sample mass is retained. The diameters D_{84} and D_{16} represent the diameters for which *15.9%* and *84.1%* of the sample mass is retained. The *standard deviation* of the logarithms of the diameters is given by:

$$\sigma = \sqrt{\frac{D_{84}}{D_{16}}} \qquad \textbf{[Eq. 108]}$$

If the diameters of the sediment sample do not follow a log-normal distribution, the *geometric mean* can be used as a representative value of the sediment size:

$$D_g = \sqrt{D_{84} \cdot D_{16}} \qquad \textbf{[Eq. 109]}$$

Example 43 - Sediment size data analysis

Sediment size data from a sieve analysis can be used to determine the characteristic diameters for a sediment sample. This information is useful in the prediction of sediment transport discharges in rivers and streams. The table below shows the results from a sieve analysis of a sample of sandy sediment:

sieve opening (mm)	0.495	0.417	0.351	0.295	0.246	0.208	0.175	0.147	0.124	0.104	0.088	0.074	pan
amount retained (g)	0.85	1.56	3.08	3.82	5.35	5.69	4.31	5.06	2.37	1.16	0.21	0.12	0.04

Using a *probability-logarithmic* plot, determine the values of D_{16}, D_{50}, and D_{84}, calculate the standard deviation of the logarithms, and the geometric mean.

From the table given above, we can produce the following table to determine the percentage finer than the given sieve size.

sieve opening (mm)	weight retained (g)	percentage weight retained	cumulative % weight retained	percent finer
0.495	0.85	2.53	2.53	97.470
0.417	1.56	4.64	7.17	92.830
0.351	3.08	9.16	16.33	83.670
0.295	3.82	11.36	27.69	72.310
0.246	5.35	15.91	43.6	56.400
0.208	5.69	16.92	60.52	39.480
0.175	4.31	12.82	73.34	26.660
0.147	5.06	15.05	88.39	11.610
0.124	2.37	7.05	95.44	4.560
0.104	1.16	3.45	98.89	1.110
0.088	0.21	0.62	99.51	0.490
0.074	0.12	0.36	99.87	0.130
pan	0.04	0.12	100.00	0.000
sum	33.62	100.00	-	-

The *probability-logarithmic* plot is shown below. The *x* axis denotes the sieve opening, in *mm*, in the logarithmic scale, while the *y* axis displays the *percentage finer* in the probability scale. The plot also shows the lines corresponding to *84.1%-, 50%-*, and *15.9%-finer* which are used to determine the values of D_{16}, D_{50}, and D_{84}. These values are:

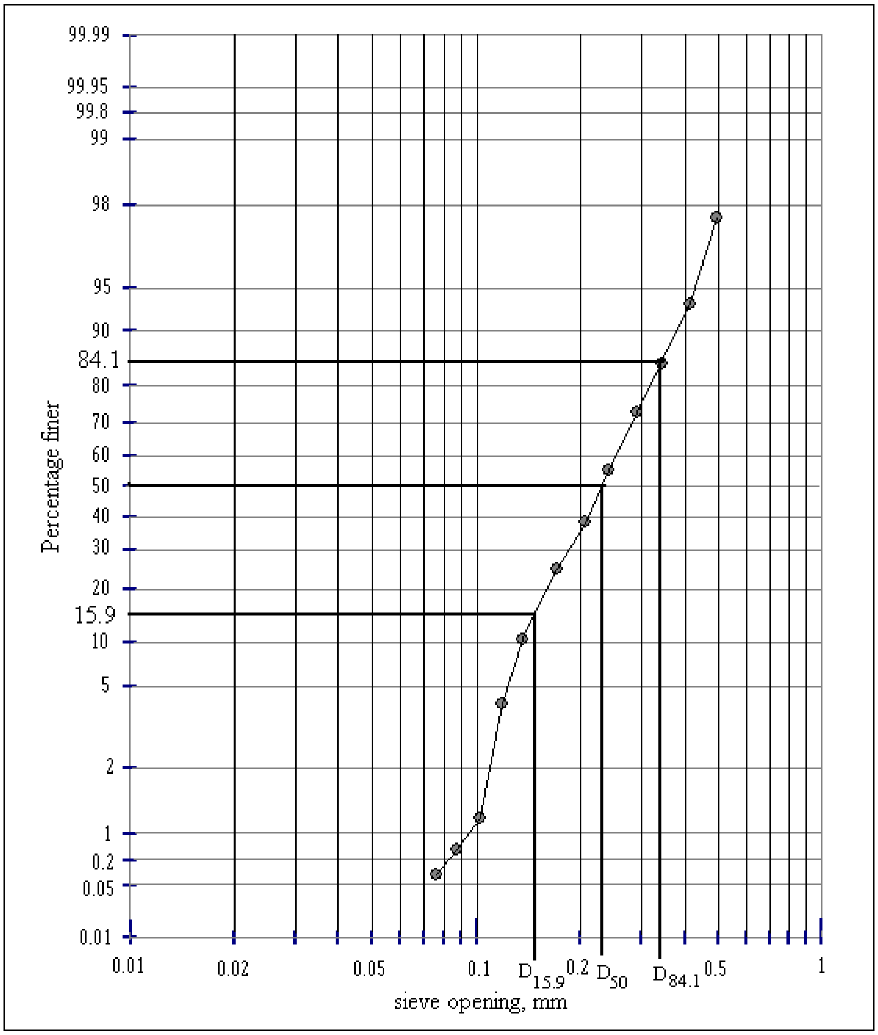
$$D_{16} = 0.15\ mm,\ D_{50} = 0.23\ mm,\ D_{84} = 0.34\ mm$$

The standard deviation of the logarithms of the diameters is:

$$\sigma = \sqrt{\frac{D_{84}}{D_{16}}} = \sqrt{\frac{0.34mm}{0.15mm}} = 1.505$$

The geometric mean of the diameters is:

$$D_g = \sqrt{D_{84} \cdot D_{16}} = \sqrt{0.34mm \times 0.15mm} = 0.225mm$$



In the analysis of suspended sediment, i.e., those sediment particles that are entrained in the moving fluid, it's important to consider the settling velocity of the sediments. Theoretical analysis of settling velocities indicate that the terminal velocity *w* of a spherical particle of diameter *D* is given by

$$w = \sqrt{\frac{4gD}{3C_D} \cdot \left(\frac{\rho_s - \rho}{\rho} \right)} = \sqrt{\frac{4gD}{3C_D} \cdot (S_s - 1)} \qquad \textbf{[Eq. 110]}$$

Where C_D is the drag coefficient, *g* is the acceleration of gravity, and ρ and ρ_s are the densities of the fluid and of the solid sediment particles, respectively. A typical value used for the specific density (gravity) of the solid sediment particles (sand) is $S_s = \rho_s / \rho = 2.65$. The drag coefficient is related to the *Reynolds number* of the particle, namely:

$$\text{Re} = \frac{w \cdot D}{\nu} \qquad \textbf{[Eq. 111]}$$

where ν is the viscosity of water.

For laminar flow, i.e., $Re < 2$:

$$C_D = \frac{24}{\text{Re}} \cdot \left(1 + \frac{3}{16}\text{Re} - \frac{19}{1280}\text{Re}^2 + \frac{71}{20480}\text{Re}^3 + \ldots\right) \qquad \textbf{[Eq. 112]}$$

The approximation $C_D = 24/Re$, applies for $Re < 0.5$. For turbulent flow, with $Re < 800$, the drag coefficient on a spherical particle is approximated by:

$$C_D = \frac{24}{\text{Re}} \cdot \left(1 + 0.150 \cdot \text{Re}^{0.687}\right) \qquad \textbf{[Eq. 113]}$$

Example 44 - Determining settling velocity of spherical sediments

Determine the settling velocity of a spherical sediment particle with a diameter $D = 0.23$ $mm = 7.54\times10^{-4}\ ft$, in water at $60\ ^oF$ ($\nu = 1.217\times10^{-5}\ ft^2/s$). Use $S_s = 2.65$ for the specific gravity of the solid particles. (This type of information is useful for designing sediment settling basins).

Determining the settling velocity, *w*, is a trial and error solution, since the settling velocity is related to a drag coefficient, which is related to the *Reynolds number*, which is, in turn, related to the settling velocity (the parameter to be calculated).

For efficient calculations, an iterative procedure involving the above equations 110 thru 113 may be programmed in a spreadsheet to obtain the settling velocity, $w = 0.0929\ ft/s$, with $Re = 5.76$, (in the turbulent regime).

The results from the previous example apply to spherical particles only. Natural sediments may have other shapes; therefore, the effect of that shape must be taken into account when calculating the settling velocity. A simple approach consists in multiplying the velocity calculated with equation 110 by a shape factor ψ:

$$w = \psi \cdot w_{sphere} = \psi \cdot \sqrt{\frac{4gD}{3C_D} \cdot \left(\frac{\rho_s - \rho}{\rho}\right)} = \psi \cdot \sqrt{\frac{4gD}{3C_D} \cdot (S_s - 1)} \qquad \textbf{[Eq. 114]}$$

Some typical values of the shape factor are provided in the table below.

Table 4. Shape factors for sediment settling velocity

Particle shape	Shape factor, ψ
sphere	*1.000*
cube-octahedron	*0.906*
octahedron	*0.846*
cube	*0.806*
tetrahedron	*0.670*

Example 45 – Determination of settling velocity for non-spherical sediments
In the previous example, a settling velocity $w_{sphere} = 0.0929\ ft/s$ for a spherical particle was found. If the particle shape is actually an octahedron, what is the settling velocity?

From the table, a shape factor $\psi = 0.846$ for an octahedron. The settling velocity for the particle is:

$$w = \psi \cdot w_{sphere} = 0.846 \times 0.0929\ ft/s = 0.0786\ ft/s$$

For additional information on sediment properties and settling velocity calculations, refer to: Raudkivi (1976), ASCE (2006), and NEH 654, Chapter 7, Basic Principles of Channel Design; Chapter 8, Threshold Channel Design; Chapter 9, Alluvial Channel Design; and Chapter 13, Sediment Impact Assessments.

0325.2 Threshold of Sediment Motion

Channels may be classified as threshold or alluvial. Sediment passes through a threshold channel with little impact on the channel boundary. An alluvial or movable-bed channel is more active, with an exchange of sediment between the channel boundary and the flow.

In a threshold channel, the applied forces of the flow are less than the threshold for movement of the boundary material. Equation 56 (section 0321) provides an expression for the average shear stress at the bed of an open channel in uniform conditions, and is repeated here:

$$\tau_o = \omega \cdot R_h \cdot S_o$$

Define the *shear velocity*, v_*, as:

$$v_* = \sqrt{\frac{\tau_o}{\rho}} \qquad \textbf{[Eq. 115]}$$

Although this is not a velocity that can be measured in the flow, the quantity defined above has the units of a velocity, and, being related to the bed shear stress, is appropriately named the shear velocity.

The bed shear stress, τ_o, and the shear velocity, v_*, (u_* is used interchangeably with v_*) can be used to define a set of parameters to analyze the threshold of sediment motion in open channels. The two parameters to consider are a *Reynolds number* based on the shear velocity:

$$\frac{v_* D}{\nu} \quad \textbf{[Eq. 116]}$$

and a dimensionless shear stress:

$$\frac{\tau_o}{(\omega_s - \omega)\cdot D} = \frac{\tau_o}{(S_s - 1)\cdot \omega \cdot D} \quad \textbf{[Eq. 117]}$$

where ν and ω ("γ" is used interchangeably with "ω") are the kinematic viscosity and specific weight of water, respectively, and D and ω_s are the diameter and the specific weight of the sediments. The specific gravity of the sediments is $S_s = \omega_s / \omega$ ($S_s = 2.65$ for sand).

NEH 654, Chapter 8 (Threshold Channel Design), describes the origin and meaning of the Shield's diagram. This diagram, shown in Figure 48, and based on the *Reynolds number* and the dimensionless shear stress (equations 116 and 117), has been extensively used for determining the threshold of sediment motion in open channel flow. For a given flow, points above the Shields's curve indicate sediment motion, whereas points below the Shield's curve would show no sediment motion.

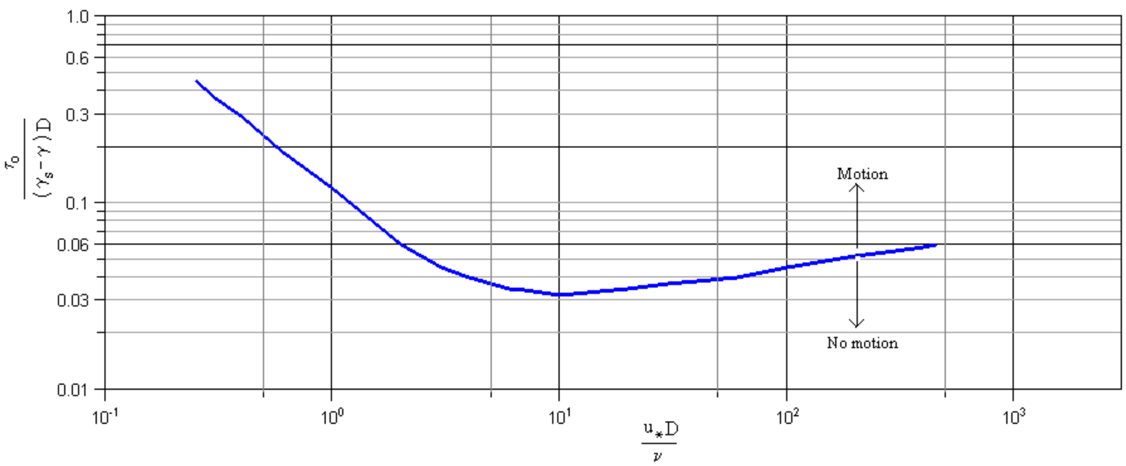


Figure 48. Shield's diagram for determining the threshold of sediment motion in open channel flow.

Example 46 – Sediment motion threshold analysis using Shield's diagram

A rectangular channel of width $b = 10\ ft$, with Manning's $n = 0.015$, and laid on a slope $S_o = 0.005$, flows at a normal depth $d_o = 0.5\ ft$. Using Shield's diagram, determine if the flow is able to move sediment of diameter $D = 0.20\ mm = 6.56\times10^{-4}\ ft$. Use $S_s = 2.65$ as the specific gravity of the sediments, and $\nu = 1.217\times10^{-5}\ ft^2/s$ for the viscosity of water.

The bed shear stress and the shear velocity are calculated as follows:

$$A = bd_o = 10\ ft \times 0.5\ ft = 5\ ft^2$$
$$P = b + 2d_o = 10\ ft + 2\times0.5\ ft = 11\ ft$$

$$R_h = A/P = 5\ ft^2/11\ ft = 0.4545\ ft$$

$$\tau_o = \omega R_h S_o = 62.4\ lb/ft^3\times0.4545\ ft\times0.005 = 0.1418\ lb/ft^2$$

$$v_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{g\omega R_h S_o}{\omega}} = \sqrt{gR_h S_o} = \sqrt{32.2\ ft/s^2 \times 0.4545\ ft \times 0.005} = 0.2705\ ft/s$$

The parameters of the Shield's diagram are calculated as follows:

$$\frac{v_* D}{\nu} = \frac{0.2705\ ft/s \times (6.56\times10^{-4}\ ft)}{1.217\times10^{-5}\ ft^2/s} = 14.6$$

$$\frac{\tau_o}{(S_s - 1)\omega D} = \frac{0.1418\ lb/ft^2}{(2.65-1)\times 62.4\ lb/ft^3 \times (6.56\times10^{-4}\ ft)} = 2.1$$

The point (14.6, 2.1) in the Shield's diagram is well above Shield's curve; therefore, sediment motion will occur for the flow specified above. Normally, sediment motion is assured if the shear stress parameter is a factor of "2" above the curve. And likewise if the shear stress parameter is less than half of the curve value, then sediment motion will not occur.

More recent work indicates that Shield's diagram does not account for the absence of stream bed forms, the sporadic entrainment of sediment particles at low shear stress, or the effects of non-uniform bed material. See NEH 654.0804(b) to calculate a refined allowable shear stress parameter.

Grass linings have been widely used to protect the erodible soil boundaries of waterways, floodways, and reservoir auxiliary spillways. An effective stress design approach is provided in the basic reference USDA AH 667. Waterway Design Tool software has been developed to design grassed waterways, following the procedures of AH 667. Refer to EFH, 650, Chapter 7, Grassed Waterways, for example designs of trapezoidal and parabolic-shaped grassed waterways, using an extensive set of design tables. Also, refer to NEH 654.0806 for more information and an example problem on the threshold design of a grass-lined channel.

0325.3 Suspended Sediment Load

Once sediment particles are moving, the finest particles may be entrained in the flow and kept in suspension by turbulent motions while being carried downstream by the bulk flow. The sediment thus transported is known as the *suspended sediment load*.

The concentration $C(y)$ of suspended sediment, in units of sediment mass per unit liquid mass, can be calculated using the following equation:

$$\frac{C(y)}{C_a} = \left(\frac{d-y}{d-a} \cdot \frac{a}{y} \right)^z \qquad \textbf{[Eq. 118]}$$

Where y is the distance from the channel bed, C_a is the concentration measured at a reference level $y = a$ close to the channel bed, d is the flow depth, and z is a parameter calculated as:

$$z = \frac{w}{\beta \cdot \kappa \cdot v_*} \qquad \textbf{[Eq. 119]}$$

In this parameter, w is the settling velocity of the sediment particles, β is the ratio of the sediment transport coefficient to the turbulent kinematic viscosity in the flow, κ is von Karman's constant (related to the viscous stress model commonly used in open channel flow), and v_* is the shear velocity. The value of β is close to 1.0 for fine sediments and decreases as the particle size increases. Von Karman's constant has been measured to be $k = 0.40$ in clear water, and lower for sediment-laden water.

If we measure the concentration at the midpoint of the depth, i.e., $C_{md} = C(d/2)$, equation 118 can be written as follows ($a = d/2$):

$$\frac{C(y)}{C_{md}} = \left(\frac{d-y}{y} \right)^z \qquad \textbf{[Eq. 120]}$$

The suspended sediment discharge G_{ss} (mass per unit volume) in a rectangular channel of width b, can be calculated by integrating the product $C(y) \cdot v(y)$ over the depth of flow, with $v(y)$ being the flow velocity distribution in the vertical:

$$G_{ss} = \int_A C(y) \cdot v(y) \cdot dA = \int_A C(y) \cdot v(y) \cdot b \cdot dy \qquad \textbf{[Eq. 121]}$$

A typical velocity distribution in turbulent open channel flow is the logarithmic distribution given by:

$$v(y) = V + \frac{v_*}{\kappa} \cdot \left[1 + \ln\left(\frac{y}{d} \right) \right] \quad \textbf{[Eq. 122]}$$

where V is the mean flow velocity. The point near the channel bed where the velocity becomes zero is located at a distance y_o from the channel bed. The value of y_o can be found from the above equation by making $v(y_o) = 0$, resulting in:

$$y_o = d \cdot \exp\left[-\left(\frac{\kappa \cdot V}{v_*} + 1 \right) \right] \quad \textbf{[Eq. 123]}$$

This value becomes the lower limit for the integral of equation 121, the upper limit being the depth of flow d. The suspended sediment discharge per unit width can be calculated as:

$$g_{ss} = \frac{G_{ss}}{b} = \int_{y_o}^{d} C(y) \cdot v(y) \cdot dy = \int_{y_o}^{d} C_{md} \cdot \left(\frac{d-y}{y} \right)^{z} \cdot \left(V + \frac{v_*}{\kappa} \cdot \left[1 + \ln\left(\frac{y}{d} \right) \right] \right) \cdot dy \quad \textbf{[Eq. 124]}$$

A dimensionless unit sediment discharge can be obtained by dividing g_{ss} by the product qC_{md}, where $q = Vd$ is the unit water discharge (or discharge per unit width). This dimensionless unit sediment discharge can be written as:

$$g_{ss}^{*} = \frac{g_{ss}}{q \cdot C_{md}} = \left(1 + \frac{v_*}{\kappa \cdot V} \right) \cdot I_1(z, \eta_o) + \frac{v_*}{\kappa \cdot V} \cdot I_2(z, \eta_o) \quad \textbf{[Eq. 125]}$$

Where $\eta_o = y_o/d$, and the integrals $I_1(z, \eta_o)$ and $I_2(z, \eta_o)$, are calculated as:

$$I_1(z, \eta_o) = \int_{\eta_o}^{1} \left(\frac{1-\eta}{\eta} \right)^{z} \cdot d\eta \quad \textbf{[Eq. 126]}$$

and

$$I_2(z, \eta_o) = \int_{\eta_o}^{1} \left(\frac{1-\eta}{\eta} \right)^{z} \cdot \ln(\eta) \cdot d\eta \quad \textbf{[Eq. 127]}$$

Example 47 – Suspended sediment discharge calculation

Consider a rectangular stream of width $b = 20\ ft$, laid on a slope $S_o = 0.00025$, with Manning's $n = 0.025$, flows at a normal depth $d_o = d = 5.0\ ft$. The mid-depth concentration is measured to be $C_{md} = 0.05\ gm/lt = 0.00312\ lb/ft^3$. The settling velocity of the particles has been calculated to be $w = 0.12\ ft/s$. Calculate the suspended sediment discharge G_{ss}, assuming that the parameter $\beta = 0.88$.

Proceeding to calculate the mean velocity and shear velocity:

$$A = bd_o = 20\ ft \times 5\ ft = 100\ ft^2$$

$$P = b + 2d_o = 20\ ft + 2{\times}5\ ft = 30\ ft$$

$$R_h = A/P = 100\ ft^2/30\ ft = 3.33\ \ ft$$

$$V = \frac{C_u}{n} \cdot R_h^{2/3} \cdot \sqrt{S_o} = \frac{1.486}{0.025} \cdot (3.33)^{2/3} \cdot \sqrt{0.00025} = 2.097\ ft/s$$

$$v_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{\frac{g\omega R_h S_o}{\omega}} = \sqrt{gR_h S_o} = \sqrt{32.2\ ft/s^2 \times 3.33\ ft \times 0.00025} = 0.1638\ ft/s$$

The lower limit in the integral of equation 121 is calculated with $\kappa = 0.40$ as:

$$y_o = d \cdot \exp\left[-\left(\frac{\kappa \cdot V}{v_*} + 1\right)\right] = 5\ ft \cdot \exp\left[-\left(\frac{0.4 \cdot 2.097\ ft/s}{0.1638\ ft/s} + 1\right)\right] = 0.0109\ ft$$

while the lower limit of the integrals in equations 126 and 127 is:

$$\eta_o = y_o/d = 0.0109ft/5ft = 0.00219$$

The parameter z from equation 119 is calculated with $\kappa = 0.40$ as follows:

$$z = \frac{w}{\beta \cdot \kappa \cdot v_*} = \frac{0.12\ ft/s}{0.88 \times 0.40 \times 0.1638\ ft/s} = 2.081$$

The integrals in equations 126 and 127 can be calculated by using numerical integration in a spreadsheet, which calculates the integrals, I_1 = 677.1623 and I_2 = -3553.72.

The dimensionless unit suspended sediment discharge is calculated with equation 125:

$$g_{ss}^* = \left(1 + \frac{0.1638}{0.4 \cdot 2.097}\right) \cdot 677.1623 + \frac{0.1638}{0.4 \cdot 2.097} \cdot -3553.72 = 115.52$$

The unit suspended sediment discharge is:

$$g_{ss} = g_{ss}^* \cdot q \cdot C_{md} = 115.52 \cdot \frac{2.097 \cdot 100}{20} \cdot 0.00312 = 3.78\ \ lb/s/\ ft$$

The suspended sediment discharge is:

$$G_{ss} = g_{ss} \cdot 20 = 75.58\ lb/s$$

0325.4 Bed Sediment Load

Coarser sediment particles carried by a stream may move by rolling and saltation along the bed of the channel constituting what is known as *bed sediment load*. Refer to NEH 654, Chapter 9, for bed sediment load formulae and where to apply. Using the *Meyer-Peter-Muller* formula due to its simplicity:

$$g_B = 8 \cdot \omega \cdot \sqrt{g \cdot D^3} \cdot \left(\frac{S_s}{S_s - 1} \right) \cdot \left[\frac{R_h \cdot S_o}{D} \cdot \left(\frac{k}{k'} \right)^{3/2} - 0.047 \cdot (S_s - 1) \right]^{3/2} \quad \textbf{[Eq. 128]}$$

In this formula, g_B is the bed sediment discharge (mass per unit time) per unit width of a rectangular channel, ω is the specific weight of water, g is the acceleration of gravity, $D = D_{50}$ is the sediment particle diameter, S_s is the specific gravity of sediments (e.g., $S_s = 2.65$ for sand), R_h is the hydraulic radius (which can be taken as $R_h = d$, the depth of flow, for a wide channel), and S_o is the bed slope. The coefficients *k* and *k'* are *Stickler coefficients* defined by the following equations:

$$V = kR_h^{2/3}\sqrt{S_o} = \frac{C_u}{n} R_h^{2/3}\sqrt{S_o} \quad \textbf{[Eq. 129]}$$

$$V = k' R_h^{2/3}\sqrt{S'} = \frac{C_u}{n'} R_h^{2/3}\sqrt{S'} \quad \textbf{[Eq. 130]}$$

The first expression for *V* is Manning's equation with the coefficient *k* accounting for the overall channel resistance, namely, bed form resistance (e.g., dunes, bars) and grain friction resistance. This velocity is defined in terms of the bed slope S_o. The second expression for *V* is Manning's equation with the coefficient *k'* accounting only for the grain friction resistance. This equation uses a slope *S'* that corresponds to the grain friction resistance. Thus, *S'* can be defined as a grain-friction energy slope.

The ratio *k/k'*, that appears in the Meyer-Peter-Muller equation, can take values between *0.5* and *1.0*, with *k/k' = 1.0* when no bed forms are present, and *k/k' = 0.5* when strong bed forms are present. The coefficient *k'* can be calculated, using metric units, as:

$$k' = \frac{26\, m^{1/3} / s}{D_{90}^{1/6}} \quad \textbf{[Eq. 131]}$$

where D_{90} is the diameter of bed sediments for which 90% of the material is finer. In the English system of units the equation to use is:

$$k' = \frac{38.63\, ft^{1/3} / s}{D_{90}^{1/6}} \quad \textbf{[Eq. 132]}$$

Example 48 – Bed sediment load rate calculation

A rectangular stream of width $b = 10\ ft$, laid on a slope $S_o = 0.00025$, with Manning's $n = 0.025$, flows at a normal depth $d_o = d = 1.5\ ft$. The sediment particles specific gravity is $S_s = 2.65$, and the diameters $D_{50} = D = 0.20\ mm = 0.20\times10^{-3}\ m = 6.56\times10^{-4}\ ft$, and $D_{90} = 0.28\ mm = 0.28\times10^{-3}\ m$. Determine the bed sediment load rate (*lb/s*) for this stream.

The calculations proceed as follows:

$$A = bd_o = 10\ ft \times 1.5\ ft = 15\ ft^2$$

$$P = b + 2d_o = 10\ ft + 2\times1.5\ ft = 13\ ft$$

$$R_h = A/P = 15\ ft^2/13\ ft = 1.15\ \ ft$$

$$k' = \frac{26}{D_{90}^{1/6}} = \frac{26}{(0.28\times10^{-3})^{1/6}} = 101.65$$

$$k = \frac{C_u}{n} = \frac{1.486}{0.025} = 59.44$$

$$k/k' = 0.5847.$$

$$g_B = 8\cdot\omega\cdot\sqrt{g\cdot D^3}\cdot\left(\frac{S_s}{S_s-1}\right)\cdot\left[\frac{R_h\cdot S_o}{D}\cdot\left(\frac{k}{k'}\right)^{3/2} - 0.047\cdot(S_s-1)\right]^{3/2} = K_1K_2$$

with (note that K_1 and K_2 are used only to aid computation)

$$K_1 = 8\cdot62.4\,lb/\ ft^3\cdot\sqrt{32.2\,ft/s^2\cdot(6.56\times10^{-4}\ ft)^3}\cdot\left(\frac{2.65}{1.65}\right) = 0.07644\,lb/\ ft/s$$

$$K_2 = \left[\frac{1.15\,ft\cdot0.00025}{6.56\times10^{-4}\ ft}\cdot(0.5847)^{3/2} - 0.047\cdot(2.65-1)\right]^{3/2} = 0.0407$$

$$g_B = K_1K_2 = 0.07644\ lb/ft/s \times 0.0407 = 0.0031\ lb/ft/s$$

For the width $b = 10\ ft$, the bed sediment load rate is:

$$G_B = g_Bb = 0.0031\ lb/ft/s \times 10\ ft = 0.031\ lb/s$$

0325.5 Scour and Deposition in Channels

A channel in equilibrium, from the point of view of sediment transport, is one which is not degrading (losing bed material) nor aggrading (gaining bed material). A decrease in the sediment supply to the channel may cause degradation. For example, if a dam has been built in a stream that typically carries sediment, the water downstream from the dam may contain much less sediment than before and, most likely, would pick up local materials to make up for the loss. On the other hand, an increase in the sediment supply to a channel may cause aggradation as the channel may not be able to carry the additional supply. Refer to NEH 654, Chapter 13, for additional information on local scour calculations.

Consider a degrading channel, of infinite length, as consequence of the building of a dam. This situation is depicted in the following figure.

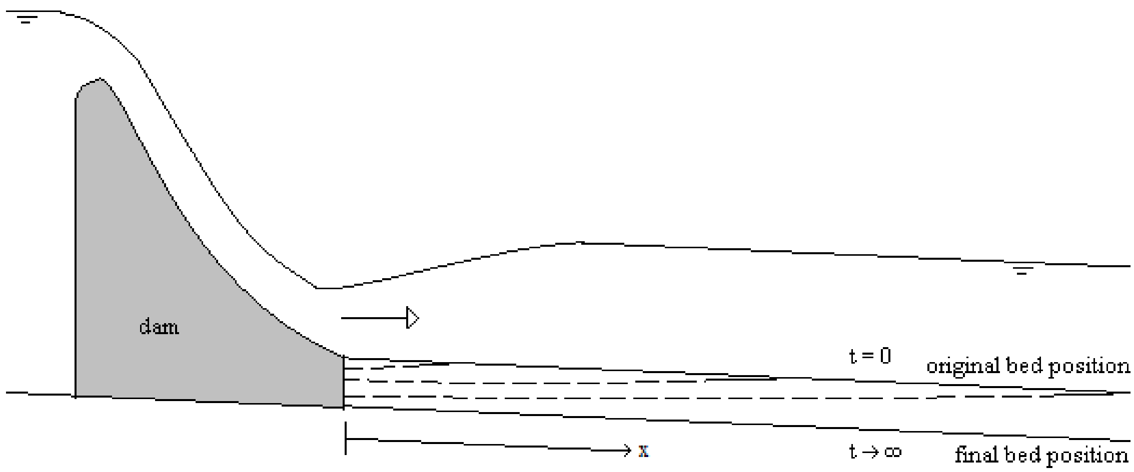


Figure 49. Channel bed degradation downstream of a dam due to reduction in sediment supply.

Let S_o *be* the slope of the original bed position and S_∞ be the slope of the final bed position. The solution that describes the change in the bed elevation with respect to time t and space x, measured from the dam foot, is given by (see Raudkivi, 1976):

$$z(x,t) = (S_\infty - S_o) \cdot \left[2\sqrt{\frac{K \cdot t}{\pi}} \cdot \exp\left(-\frac{x^2}{4Kt} \right) - x \cdot erfc\left(\frac{x}{2\sqrt{Kt}} \right) \right] \quad \textbf{[Eq. 133]}$$

Where K is a parameter defined in terms of an equation of flow (the Chezy equation) and the sediment discharge expression (equation 136), and *erfc* is the complementary error function defined as:

$$erfc = 1 - erf(\theta) = 1 - \int_0^{\vartheta} \exp(-\xi) d\xi \quad \textbf{[Eq. 134]}$$

and *erf* is the error function.

The Chezy equation for the flow:

$$V = C\sqrt{d \cdot S_f} \qquad \textbf{[Eq. 135]}$$

Where C is the Chezy coefficient, d is the flow depth (the equation is given for a wide channel), and S_f is the energy slope (slope of the energy line).

The sediment discharge expression is given in terms of the flow velocity V as an empirical equation:

$$g_s = a \cdot V^b \qquad \textbf{[Eq. 136]}$$

where g_s is the unit sediment discharge (sediment discharge per unit width), and a and b are constant values. The parameter K in equation 133 is calculated as:

$$K = \frac{C^2 b q a^{3/2} g_s^{1-3/b}}{3 \cdot (1-\lambda)} \qquad \textbf{[Eq. 137]}$$

where $q = Vd$ is the water discharge per unit width, λ is the porosity of the bed sediments (typical value, $\lambda = 0.40$), g_s, a, and b are defined in equation 136, and C is the Chezy coefficient (see equation 135). *Porosity* of soils or sediments is defined as the ratio of the volume of voids in the material to the total volume (solids + voids).

Defining the amount of degradation about the dam as:

$$z_o = 2 \cdot (S_\infty - S_o)\sqrt{\frac{Kt}{\pi}} \qquad \textbf{[Eq. 138]}$$

With this definition, equation 133 can be written to give a dimensionless bed elevation:

$$\frac{z}{z_o} = \exp\left(-\frac{x^2}{4Kt}\right) - \frac{x}{2}\sqrt{\frac{\pi}{Kt}} \cdot erfc\left(\frac{x}{2\sqrt{Kt}}\right) \qquad \textbf{[Eq. 139]}$$

Defining the initial and final sediment supply as:

$$g_{so} = K \cdot (1-\lambda) \cdot S_o \qquad \textbf{[Eq. 140]}$$

and

$$g_{s\infty} = K \cdot (1-\lambda) \cdot S_\infty \qquad \textbf{[Eq. 141]}$$

The parameter z_o can be written as:

$$z_o = \frac{2 \cdot (g_{s\infty} - g_{so})}{1-\lambda}\sqrt{\frac{t}{\pi K}} \qquad \textbf{[Eq. 142]}$$

Notice that the solution provided by equation 133, applies equally to degradation ($g_{so}>g_{s\infty}$) or aggradation ($g_{so}<g_{s\infty}$).

Example 49 – Bed aggradation

A laboratory flume is set at a slope $S_o = 0.00356$ and water flows at a uniform depth $d_o = 2\ inches = 0.167\ ft$, and a velocity $V = 1.31\ ft/s$. Sediment is supplied into the flow at a rate $g_{so} = 1.598\times10^{-4}\ ft^2/s$ until the channel reaches equilibrium. The estimated value of the parameter K is $K = 0.0748\ ft^2/s$, and the porosity of sediments is $\lambda = 0.40$. Without changing any of the flow conditions, the sediment supply rate is increased to $g_{s\infty} = 7.99\times10^{-4}\ ft^2/s$, so that the channel bed starts aggrading. (a) Compute the maximum amount of aggradation after $t = 1\ hr = 3600\ s$, and (b) compute the amount of aggradation at a point $x = 2.0\ ft$ downstream of the point of sediment injection, after $t = 1\ hr = 3600\ s$. Assume that the channel is of infinite length.

The maximum amount of aggradation at any given time occurs at position $x = 0$, and is given by equation 142:

$$z_o = \frac{2\cdot(g_{s\infty} - g_{so})}{1-\lambda}\sqrt{\frac{t}{\pi K}}$$

$$z_o = \frac{2\cdot(7.99\times10^{-4}\ ft^2/s - 1.598\times10^{-4}\ ft^2/s)}{1-0.4}\sqrt{\frac{3600s}{3.1416\times0.0748\,ft^2/s}} = 0.2637\,ft = 3.16\,in$$

To find the aggradation elevation for $x = 2\ ft$ and $t = 3600\ hr$, use equation 139:

$$z = z_o\cdot\left(\exp\left(-\frac{x^2}{4Kt}\right) - \frac{x}{2}\sqrt{\frac{\pi}{Kt}}\cdot erfc\left(\frac{x}{2\sqrt{Kt}}\right)\right)$$

The value of the function *erfc* may be found by use of an appropriate spreadsheet. The argument of the function for this example is:

$$\frac{x}{2\sqrt{Kt}} = \frac{2.0\,ft}{2\sqrt{0.0748\,ft^2/s\times3600s}} = 0.0609$$

The spreadsheet gives $erfc(0.0609) = 0.931367$.

The required aggradation elevation is:

$$z = 0.2637\,ft\cdot\left(\exp\left(-\frac{(2.0\,ft)^2}{4\times0.0748\,ft^2/s\times3600s}\right) - \frac{2.0\,ft}{2}\sqrt{\frac{3.1416}{0.0748\,ft^2/s\times3600s}}\cdot0.931367\right)$$

$$z = 0.2362\ ft = 2.83\ in$$

Example 50 – Bed degradation downstream of dam.

The initial slope of a river channel is $S_o = 0.0094$. It is estimated that, after construction of a dam, the resulting slope will be $S_\infty = 0.0020$. The parameter K is estimated to be $10750\ ft^2/day$. The initial bed elevation at the dam site is $z_s = 1500\ ft$ above mean sea level. Compute the expected bed elevation at a number of points along the stream channel after, (a) two months (60 days), and (b) 2 years (730 days) of the construction of the dam.

The elevation at the dam site for $t = 60\ days$, is calculated as:

$$z_o = 2\cdot(S_\infty - S_o)\sqrt{\frac{Kt}{\pi}} = 2\cdot(0.0020 - 0.0094)\cdot\sqrt{\frac{10750\,ft^2/day\times 60day}{3.1416}} = -6.71\,ft$$

And for $t = 730\ days$, the elevation z_o is calculated as:

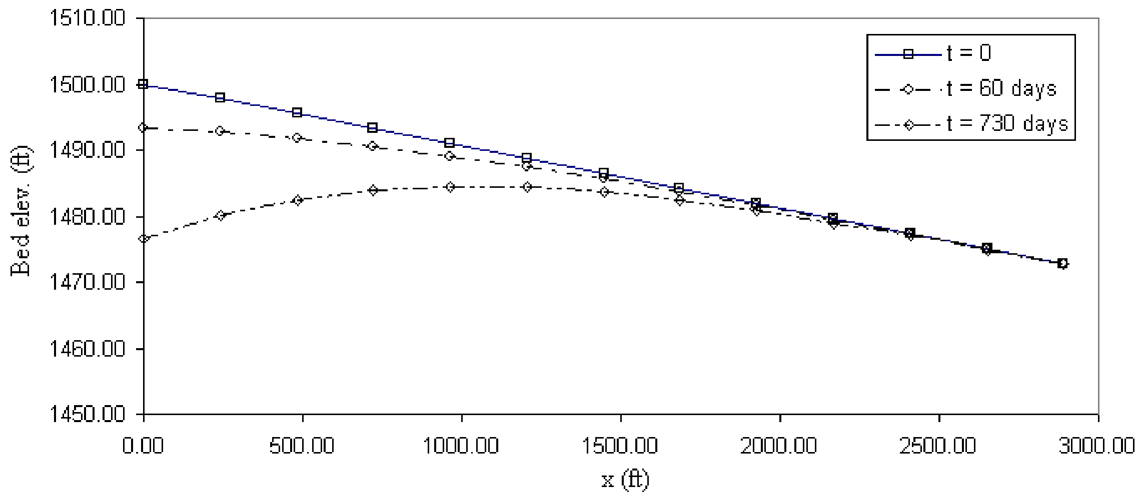
$$z_o = 2\cdot(S_\infty - S_o)\sqrt{\frac{Kt}{\pi}} = 2\cdot(0.0020 - 0.0094)\cdot\sqrt{\frac{10750\,ft^2/day\times 730day}{3.1416}} = -23.39\,ft$$

In the following table and plot, the degradation elevations z for $t = 0$, $t = 60\ day$, and $t = 730\ day$, for a number of values of "x" are presented. The bed elevation is calculated as follows:

$$Bed\ elevation(x) = Bed\ elevation(x{=}0) + z(x) - S_o\cdot x$$

The calculations and plot of the data were performed with an appropriate spreadsheet.

			t = 0	Bed	t = 60 d	Bed	t = 730 d	Bed
$x/2(Kt)^{1/2}$	z/z_o	x(ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)
0.00	1.0000	0.00	0.00	1500.00	-6.71	1493.29	-23.39	1476.61
0.15	0.7565	240.94	0.00	1497.74	-5.07	1492.66	-17.70	1480.04
0.30	0.5569	481.87	0.00	1495.47	-3.73	1491.74	-13.03	1482.44
0.45	0.3983	722.81	0.00	1493.21	-2.67	1490.53	-9.32	1483.89
0.60	0.2764	963.74	0.00	1490.94	-1.85	1489.09	-6.47	1484.48
0.75	0.1858	1204.68	0.00	1488.68	-1.25	1487.43	-4.35	1484.33
0.90	0.1209	1445.61	0.00	1486.41	-0.81	1485.60	-2.83	1483.58
1.05	0.0760	1686.55	0.00	1484.15	-0.51	1483.64	-1.78	1482.37
1.20	0.0462	1927.49	0.00	1481.88	-0.31	1481.57	-1.08	1480.80
1.35	0.0271	2168.42	0.00	1479.62	-0.18	1479.44	-0.63	1478.98
1.50	0.0153	2409.36	0.00	1477.35	-0.10	1477.25	-0.36	1476.99
1.65	0.0083	2650.29	0.00	1475.09	-0.06	1475.03	-0.19	1474.89
1.80	0.0044	2891.23	0.00	1472.82	-0.03	1472.79	-0.10	1472.72



In some instances, a control on the downstream end of a channel of length L may exist such that the bed elevation at the downstream end remains constant even as the channel aggrades or degrades. If the starting bed slope is S_o and the ending bed slope is S_∞, the elevation of the channel bed $z(x,t)$ is given by the following equation (see Raudkivi, 1976):

$$z(x,t) = S_\infty \cdot (L-x)$$

$$+\frac{8\cdot L\cdot (S_o - S_\infty)}{\pi^2}\cdot\sum_{k=1}^{\infty}\frac{1}{(2k-1)^2}\exp\left(-\frac{(2k-1)^2\pi^2 Kt}{4L^2}\right)\cdot\cos\left(\frac{(2k-1)\pi x}{2L}\right) \quad \textbf{[Eq. 143]}$$

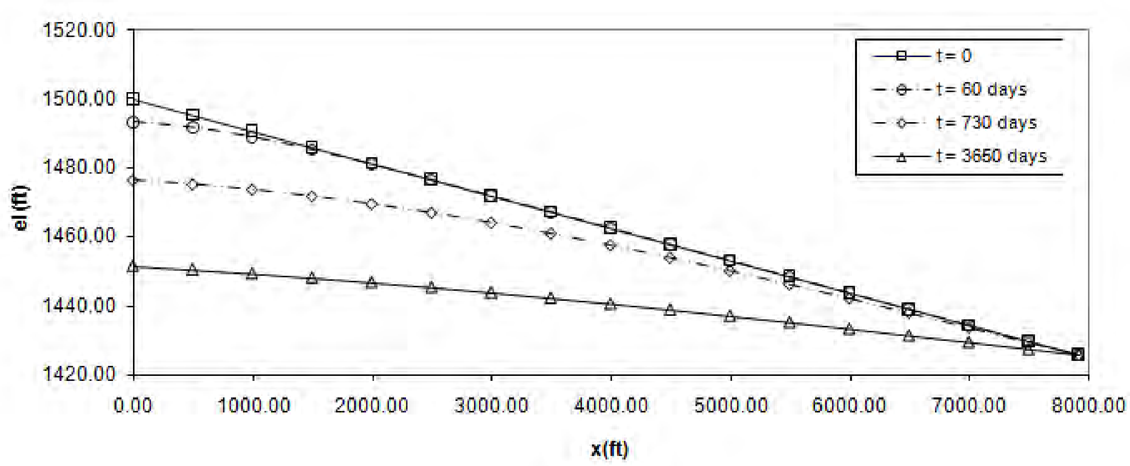
The coefficient K is calculated as in equation 137.

Example 51 – Bed degradation with constant downstream elevation

This example uses the same data as in the previous example, i.e., $S_o = 0.0094$, $S_\infty = 0.0020$, and $K = 10750\ ft^2/day$; however, a control of constant elevation occurs at the downstream end of a channel length $L = 1.5\ mi = 7920\ ft$. The initial bed elevation at the dam site is $z_s = 1500\ ft$. Compute the expected bed elevation at a number of points along the river channel after, (a) two months (60 days), (b) 2 years (730 days), and (c) 10 years (3650 days) of the construction of the dam. (Note: the value of S_∞ can be estimated from the terrain geometry and selected so as to produce a channel with a mild slope.)

The calculations and plot of the data, shown below, were performed with an appropriate spreadsheet.

time (d):	0	Bed	60	Bed	730	Bed	3650	Bed
x(ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)	z(ft)	elev (ft)
0.00	74.44	1500.00	67.74	1493.30	51.06	1476.62	25.99	1451.55
500.0	69.75	1495.31	66.10	1491.66	49.87	1475.43	24.94	1450.50
1000.0	65.05	1490.61	63.30	1488.86	48.32	1473.88	23.79	1449.35
1500.0	60.35	1485.91	59.62	1485.18	46.40	1471.96	22.54	1448.10
2000.0	55.65	1481.21	55.38	1480.94	44.14	1469.70	21.20	1446.76
2500.0	50.95	1476.51	50.87	1476.43	41.55	1467.11	19.77	1445.33
3000.0	46.25	1471.81	46.23	1471.79	38.66	1464.22	18.25	1443.81
3500.0	41.55	1467.11	41.54	1467.10	35.49	1461.05	16.64	1442.20
4000.0	36.85	1462.41	36.85	1462.41	32.08	1457.64	14.96	1440.52
4500.0	32.15	1457.71	32.15	1457.71	28.44	1454.00	13.21	1438.77
5000.0	27.45	1453.01	27.45	1453.01	24.61	1450.17	11.39	1436.95
5500.0	22.75	1448.31	22.75	1448.31	20.63	1446.19	9.53	1435.09
6000.0	18.05	1443.61	18.05	1443.61	16.51	1442.07	7.61	1433.17
6500.0	13.35	1438.91	13.35	1438.91	12.29	1437.85	5.66	1431.22
7000.0	8.65	1434.21	8.65	1434.21	8.00	1433.56	3.68	1429.24
7500.0	3.95	1429.51	3.95	1429.51	3.66	1429.22	1.68	1427.24
7920.0	0.00	1425.56	0.00	1425.56	0.00	1425.56	0.00	1425.56



0330 Pipe Flow
The principles of pipe flow apply to the hydraulics of such structures as culverts, drop inlets, siphons, and various types of pipelines.

Consider Figure 24 which shows the hydraulic grade line, energy line and the energy heads at two points of the flow. Most pipe flow occurs through constant-diameter pipe, thus, making the velocity heads at sections (1) and (2) the same ($V_1 = V_2 = V$). This in turn means that the energy line (E.L.) and hydraulic grade line (H.G.L.) are parallel. The energy equation for constant-diameter pipe is written as:

$$z_1 + \frac{p_1}{\omega} = z_2 + \frac{p_2}{\omega} + h_f \quad \textbf{[Eq. 144]}$$

Where h_f represents the energy head losses between sections (1) and (2) in Figure 24. Because the energy head losses are due to the friction between the moving fluid mass and the walls of the pipe, h_f is also referred to as the *friction losses*. Friction losses may be calculated by measuring the piezometric heads,

$h_1 = z_1 + \frac{p_1}{\omega}$, and $h_2 = z_2 + \frac{p_2}{\omega}$, and taking the difference $h_f = h_1 - h_2$.

The energy slope, or slope of the energy line, is the ratio of the friction losses, h_f, to the length of the pipe, L, and may be expressed as:

$$S_f = \frac{h_f}{L} = \frac{h_1 - h_2}{L} = \frac{1}{L} \cdot \left[\left(z_1 - \frac{p_1}{\omega} \right) - \left(z_2 - \frac{p_2}{\omega} \right) \right] \qquad \textbf{[Eq. 145]}$$

Flow in pipelines occurs in two different regimes, *laminar* and *turbulent*. Laminar flow occurs at relatively small velocities, and is characterized by fluid particles moving in stable flow layers and with a parabolic velocity distribution. Increasing the flow velocity sufficiently causes instabilities of the flow layers which, in turn, cause the flow to become turbulent. In turbulent flow the fluid particles no longer move in stable layers, but rather clump together to form different-sized *eddies*. Turbulent flow is characterized by a more uniform velocity distribution across the pipe typically described mathematically by logarithmic or power-law functions.

0331 Friction Loss Methods

Calculation of friction losses is an important step in analyzing flow in pipes, and in the selection of pipe sizes for specific applications. The calculation of friction losses in pipe can be performed through the use of a number of methods such as Manning's equation, the Darcy-Weisbach equation, or the Hazen-Williams formula. These methods are presented in detail next.

0331.1 Manning's Equation for Pipelines

Manning's equation for open-channel flow was presented as equation 71 in terms of the flow velocity V, the hydraulic radius R, the channel bed slope S_o, the Manning's resistance coefficient n, and a constant C_u that depends on the system of units used ($C_u = 1.0$ for the International System, and $C_u = 1.486$ for the English System). When applied to pipelines, the bed slope S_o is replaced by the energy slope $S_f = h_f/L$, while the hydraulic radius is calculated in terms of the area, A (equation 29 repeated here), and wetted perimeter, P, of a full circular cross-section of diameter D, using:

$$A = \frac{\pi D^2}{4}$$

$$P = \pi D \qquad \textbf{[Eq. 146]}$$

which results in:

$$R = \frac{A}{P} = \frac{D}{4} \quad \text{[Eq. 147]}$$

With these substitutions, the Manning's equation for a pipeline results in:

$$V = \frac{C_u}{n}\left(\frac{D}{4}\right)^{2/3}\sqrt{\frac{h_f}{L}} \quad \text{[Eq. 148]}$$

Typically, the equation is rewritten by isolating h_f:

$$h_f = \frac{4^{4/3}}{C_u^2}\cdot\frac{n^2\cdot L}{D^{4/3}}\cdot V^2 \quad \text{[Eq. 149]}$$

Equations based on the above equation (velocity) for both the International System (S.I.) and English System (E.S.) follow:

- International System (S.I.): h_f(m), C_u = 1.0, L(m), D(m), V(m/s)

$$h_f = 6.3496\cdot\frac{n^2\cdot L}{D^{4/3}}\cdot V^2 \quad \text{[Eq. 150]}$$

- English System (E.S.): h_f(ft), C_u = 1.486, L(ft), D(ft), V(ft/s, fps)

$$h_f = 2.8755\cdot\frac{n^2\cdot L}{D^{4/3}}\cdot V^2 \quad \text{[Eq. 151]}$$

Sometimes it is preferred to work with the discharge Q, instead of the flow velocity V. For circular pipelines of diameter D, the equation of continuity is written as either equation 30 or equation 31, which are reproduced below:

$$Q = VA = V\cdot\frac{\pi D^2}{4}$$

$$V = \frac{4Q}{\pi D^2}$$

Substituting equation 31 into equation148, and solving for Q:

$$Q = \frac{\pi\cdot C_u\cdot D^{8/3}}{4^{5/3}\cdot n}\cdot\sqrt{\frac{h_f}{L}} \quad \text{[Eq. 152]}$$

Or, solving for h_f:

$$h_f = \frac{4^{10/3}}{\pi^2 \cdot C_u^2} \cdot \frac{n^2 \cdot L}{D^{16/3}} \cdot Q^2 \qquad \textbf{[Eq. 153]}$$

Equations based on the above equation (discharge) for both the International System (S.I.) and English System (E.S.) follow:

- International System (S.I.): h_f(m), C_u = 1.0, L(m), D(m), Q(m^3/s)

$$h_f = 10.2936 \cdot \frac{n^2 \cdot L}{D^{16/3}} \cdot Q^2 \qquad \textbf{[Eq. 154]}$$

- English System (E.S.): h_f(ft), C_u = 1.486, L(ft), D(ft), Q(ft^3/s, cfs)

$$h_f = 4.6615 \cdot \frac{n^2 \cdot L}{D^{16/3}} \cdot Q^2 \qquad \textbf{[Eq. 155]}$$

Example 52 – Pipeline discharge calculation using Manning's equation

Determine the discharge Q that can be conveyed by a *3.0-in*-diameter corrugated-plastic pipeline if a head $h_f = 10$ *ft*, is to be dissipated in a length $L = 100$ *ft*.

Use equation 152 to calculate the discharge. The data to use are the following: $D = 3.0$ *in* $= 3.0/12 = 0.25$ *ft*, $h_f = 10$ *ft*, $L = 100$ *ft*, $C_u = 1.486$, $n = 0.015$ (from Table 5). The resulting discharge is:

$$Q = \frac{\pi \cdot C_u \cdot D^{8/3}}{4^{5/3} \cdot n} \cdot \sqrt{\frac{h_f}{L}} = \frac{(3.1416) \cdot (1.486) \cdot (0.25)^{8/3}}{4^{5/3} \cdot 0.015} \cdot \sqrt{\frac{10}{100}} = 0.24\,cfs$$

The software *USDA-NRCS Hydraulics Formula* provides for the calculation of pipe flow using Manning's equation. To activate this solution select the *Pipe Flow* tab, which produces the entry form shown in Figure 50. The formula shown in the entry form is equivalent to equation 152, but with the diameter D in inches and with local loss coefficients equal to zero (i.e., $Ke = 0$, $Kb = 0$).

Example 53 – Pipeline discharge calculation using the *USDA-NRCS Hydraulics Formula* software

Using the *Pipe Flow* tab in the *USDA-NRCS Hydraulics Formula* software, with $Ke = 0$, $Kb = 0$, determine the discharge Q conveyed by a *3-in*-diameter corrugated-plastic pipeline if a head $h_f = 10$ *ft*, is to be dissipated in a length $L = 100$ *ft*. These data are the same as for the previous example 52. The resulting discharge is 0.2 cfs.

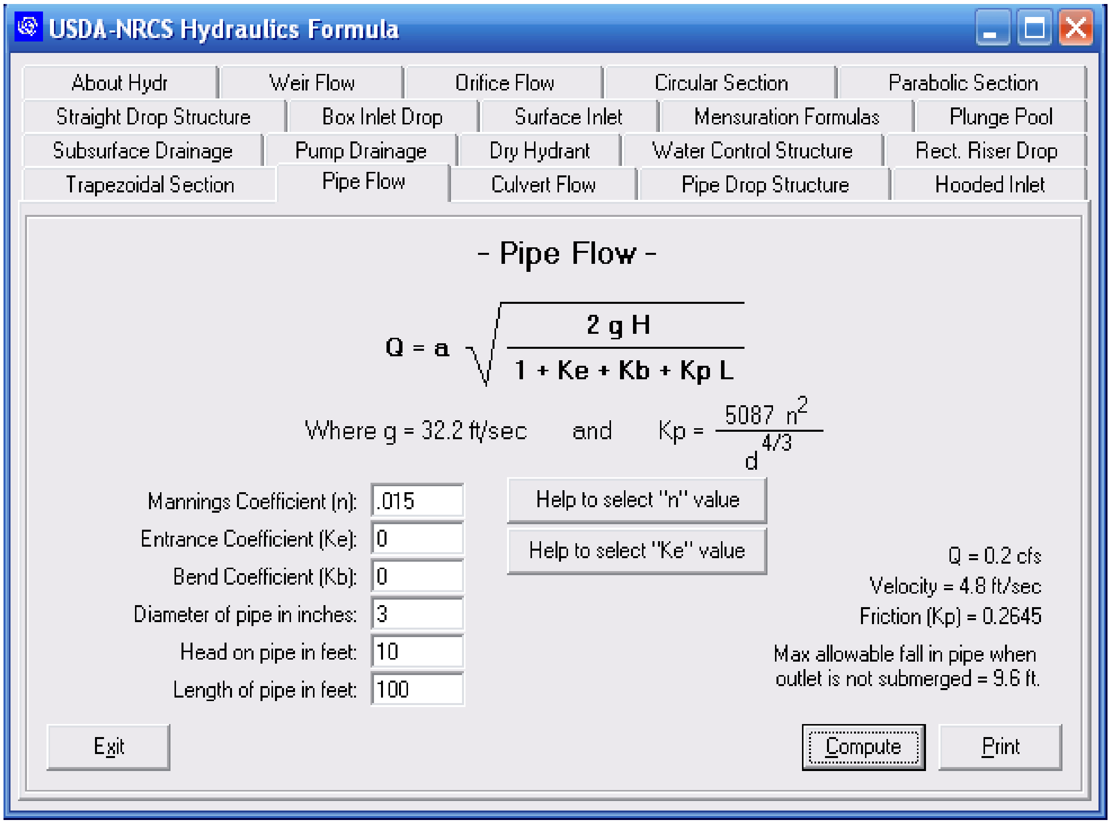


Figure 50. ***Pipe Flow*** **calculations with the** ***USDA-NRCS Hydraulics Formula*****.**

The equation for pipe flow shown in the above figure will be derived after introducing the concept of local losses in pipe (see section 0331.6).

Manning's *n* values, shown in Table 5, for various pipe materials are assembled from many hydraulic references, including Brater and King (1996), FHWA (2001), and USACOE (2008).

Table 5. Values of Manning's resistance coefficient for pipe

	Manning's *n*		
Pipe material	***Minimum***	***Design***	***Maximum***
Cast-iron, coated	0.010	0.012 - 0.014	0.014
Cast-iron, uncoated	0.011	0.013 - 0.015	0.015
Wrought iron, galvanized	0.013	0.015 - 0.017	0.017
Wrought iron, black	0.012		0.015
Steel, riveted and spiral	0.013	0.015 - 0.017	0.017
Annular corrugated metal(1)	0.021	0.021 - 0.025	0.0255
Helical corrugated metal(1)	0.013	0.015 - 0.020	0.021
Wood stave	0.010	0.012 - 0.013	0.014
Neat cement surface	0.010		0.013
Concrete	0.010	0.012 - 0.017	0.017
Vitrified sewer pipe	0.010	0.013 - 0.015	0.017
Clay, common drainage tile	0.011	0.012 - 0.014	0.017
Corrugated plastic	0.014	0.015 - 0.016	0.017
PVC		0.009 -0.011	
Smooth Interior PE		0.009 - 0.015	0.02
Aluminum		0.01	
Gated Aluminum Pipe		0.013	

(1) N-values for corrugated metal pipe vary with pipe diameter. See FHWA (2001) or USACOE (2008) to select a refined n-value

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Example 54 – Flow velocity in a helical corrugated metal pipe using Manning's equation

Determine the flow velocity in a helical corrugated metal pipe (use $n = 0.016$, from Table 5), that is *5-in* in diameter if it dissipates a head $h_f = 6.5\ ft$ in a length $L = 300\ ft$.

With $D = 5\ in = 5/12\ ft = 0.4167\ ft$, and $Cu = 1.486$, equation 148 produces the following result:

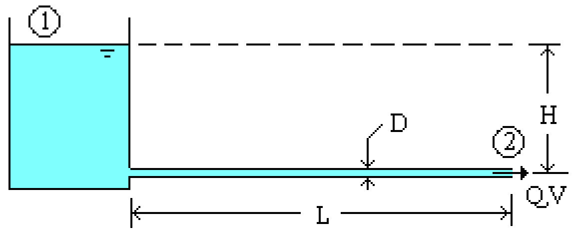
$$V = \frac{C_u}{n}\left(\frac{D}{4}\right)^{2/3}\sqrt{\frac{h_f}{L}} = \frac{1.486}{0.016}\left(\frac{0.4167}{4}\right)^{2/3}\sqrt{\frac{6.5}{300}} = 3.0\ fps$$

Example 55 – Head loss in a riveted and spiral steel pipe using Manning's equation

Determine the head loss in 1500 ft of a riveted and spiral steel pipe (use $n = 0.015$) with a 24-in (2-ft) diameter that carries a discharge of 10 cfs. Using equation 155:

$$h_f = 4.6615 \cdot \frac{n^2 \cdot L}{D^{16/3}} \cdot Q^2 = 4.6615 \cdot \frac{0.015^2 \cdot 1500}{2.0^{16/3}} \cdot 10^2 = 3.9\ ft$$

Example 56 – Velocity in pipeline draining a reservoir – Manning's equation
Consider a reservoir whose free surface is located at an elevation $z_1 = 60\ ft$, draining through a *0.5-ft*-diameter, *100-ft*-long, concrete pipe ($n = 0.012$) open to the atmosphere whose outlet is located at an elevation $z_2 = 55\ ft$. Determine the velocity in the pipeline. The system is depicted in the following figure. Entrance losses will be ignored.



Point *1* in the energy equation is at the reservoir free surface where $p_1 = 0$ and $V_1 = 0$. Point *2* is at the pipe outlet where $p_2 = 0$ and $V_2 = V$, the pipe velocity. The energy head, $H = z_1 - z_2 = 60\ ft - 55\ ft = 5\ ft$, for this case. Applying the energy equation (equation 47) between points *1* and *2*:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f$$

Using Manning's equation to represent friction losses:

$$z_2 + H + \frac{0}{\omega} + \frac{0^2}{2g} = z_2 + \frac{0}{\omega} + \frac{V^2}{2g} + 2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} \cdot V^2$$

This simplifies to:

$$H = V^2 \cdot \left(\frac{1}{2g} + \frac{2.8755 \cdot n^2 \cdot L}{D^{4/3}} \right)$$

Solving for the velocity, V, and using $H = 5\ ft$, $L = 100\ ft$, $n = 0.012$, $D = 0.5\ ft$, $g = 32.2\ ft/s^2$:

$$V = \sqrt{\frac{H}{\frac{1}{2g} + \frac{2.8755 \cdot n^2 \cdot L}{D^{4/3}}}} = \sqrt{\frac{5}{\frac{1}{2 \times 32.2} + \frac{2.8755 \cdot 0.012^2 \cdot 100}{0.5^{4/3}}}} = 6.46\ ft/s$$

0331.2 Darcy-Weisbach Equation and Friction Factor
A second method for calculating friction losses in pipes is the Darcy-Weisbach equation written, in terms of the flow velocity *V* as:

$$h_f = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} \qquad \textbf{[Eq. 156]}$$

The friction factor, *f*, is a function of a *relative roughness*, *e/D*, where *e* is known as the *absolute roughness* or *equivalent sand roughness*, and of the Reynolds number of the flow.

The Reynolds number, defined in equation 10, is repeated here:

$$\mathrm{Re} = \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

where ρ is the density (mass per unit volume) of water, μ is the *absolute* or *dynamic viscosity* of water, and ν is the *kinematic viscosity* of water, defined by equation 9, $\nu = \mu/\rho$. Values of the density and viscosity of water related to temperature are available in Exhibit 4.

The absolute roughness of a pipe material is the average height of the irregularities of the inner wall of the pipe. The first experiments on head losses in pipes were conducted in the early 20th century by coating glass pipes with uniform-size sand grains. The diameter of the sand grains was used to represent the absolute roughness of the pipe, *e*. Typical values of the absolute roughness of various pipe materials are presented in Table 6. Absolute roughness values may be found in Streeter and Wylie (1998) and other similar fluid mechanics texts.

Table 6. Absolute roughness values for pipe materials

Pipe material	***e* (*mm*)**	***e* (*ft*)**
Smooth surface (glass, plastic)	0	0
Drawn tubing, brass, lead, copper	0.0015	0.000005
Centrifugally spun cement	0.0015	0.000005
Bituminous lining	0.0015	0.000005
PVC pipe	0.002	0.0000066
Commercial steel	0.046	0.00015
Wrought iron	0.046	0.00015
Welded-steel pipe	0.046	0.00015
Asphalt-dipped cast iron	0.12	0.0004
Aluminum, with couplers	0.13	0.00043
Galvanized iron	0.15	0.0005
Cast iron	0.25	0.00085
Wood stave	0.18-0.9	0.0006-0.003
Concrete	0.3-3	0.001-0.01
Riveted steel	0.9-9	0.003-0.03

The Darcy-Weisbach friction factor has different expressions for laminar or turbulent pipe flow, described in the following section.

0331.3 Laminar and Turbulent Friction Factor Equations

Water flowing at a very low velocity, or in a very small-diameter pipe, typically flows in a *laminar* flow regime. In laminar flow the flow takes place in layers (Latin, *laminae*) that remain very stable and are easily identifiable by dye injected into the flow. As the velocity increases, conditions are reached in which the flow becomes *turbulent*. In turbulent flow, layers of flow are no longer identifiable and the flow tends to break down into eddies that facilitate mixing. Laminar flow is rare, occurring at low flow velocity or small pipe diameter. Most pipe flows of interest are turbulent.

Experiments to determine the laminar or turbulent nature of flow were carried out by Osborne Reynolds in the 19^{th} century. Reynolds identified a dimensionless parameter, now known as the *Reynolds number* (equation 10), to determine if a flow is laminar or turbulent. There is also a *transitional flow* regime in which the flow is neither laminar nor turbulent but shifts between conditions. The typical values for classifying flows according to laminar, transitional, or turbulent regimes are shown next:

- Laminar flow: $Re<2000$
- Transitional flow: $2000 < Re < 4000$
- Turbulent flow: $Re > 4000$

The range Re < 4000 that encompasses laminar and transitional flow is a small range considering that Re can take values to 10^9 or larger. Thus, turbulent flow is more likely to occur in pipelines.

Example 57 – Reynolds number used to classify pipe flow

Water is flowing in a 2-in diameter pipe at a rate of 0.08 cfs. If the water temperature is 70^oF, determine the Reynolds number of the flow and classify it as laminar, transitional, or turbulent.

The data given is *D = 2 in = 2/12 ft = 0.167 ft*, *Q = 0.08 cfs*, and from Exhibit 4 at a temperature of 70^oF the kinematic viscosity of water is $\nu = .00001059\ ft^2/s = 1.059\times10^{-5}\ ft^2/s$. The flow velocity is calculated as:

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4\times 0.08\ ft^3/s}{3.1416\times(0.167\ ft)^2} = 3.7\ ft/s$$

And the Reynolds number is:

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(3.7\ ft/s)\times(0.167\ ft)}{1.059\times10^{-5}\ ft^2/s} = 58347 \approx 5.83\times10^4$$

Since *Re > 4000*, the flow is turbulent.

In the laminar regime, the Darcy-Weisbach friction factor, *f*, is a function of the Reynolds number, given as:

$$f = \frac{64}{\mathrm{Re}} \qquad \textbf{[Eq. 157]}$$

In the turbulent regime, an equation relating the friction factor, *f*, the Reynolds number *Re*, and the relative roughness, *e/D*, is the *Colebrook-White* equation:

$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left(\frac{e}{3.7 \cdot D} + \frac{2.51}{\mathrm{Re} \cdot \sqrt{f}}\right) \qquad \textbf{[Eq. 158]}$$

The difficulty in using this equation is that it is not explicit in *f* and any solution involving this equation requires a numerical approach. To facilitate solving explicitly for *f*, a close approximation (2 – 5% error) to the *Colebrook-White* equation is provided by the *Swamee-Jain* equation:

$$f = \frac{0.25}{\log^2\left(\frac{e}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)} \qquad \textbf{[Eq. 159]}$$

In the following section, solutions of turbulent pipe flow using the Darcy-Weisbach equation are presented.

0331.4 Pipe Flow Solutions Using the Darcy-Weisbach Equation

Since the discharge, *Q*, is often known or to be calculated, it is convenient to write the Darcy-Weisbach equation in terms of the discharge as follows:

$$h_f = \frac{8 \cdot f \cdot L \cdot Q^2}{\pi^2 \cdot g \cdot D^5} \qquad \textbf{[Eq. 160]}$$

The Reynolds number *Re* can also be written in terms of the discharge *Q* as follows:

$$\mathrm{Re} = \frac{4 \cdot \rho \cdot Q}{\pi \cdot \mu \cdot D} = \frac{4 \cdot Q}{\pi \cdot \nu \cdot D} \qquad \textbf{[Eq. 161]}$$

With this definition of the Reynolds number, the Swamee-Jain equation becomes:

$$f = \frac{0.25}{\log^2\left(0.27 \cdot \frac{e}{D} + 4.62 \cdot \left(\frac{\nu D}{Q}\right)^{0.9}\right)} \quad \textbf{[Eq. 162]}$$

Combining the Darcy-Weisbach equation (equation 160) with the Swamee-Jain equation (equation 162), and solving for the discharge, Q, produces the following equation:

$$Q = -2.22\sqrt{\frac{gD^5 h_f}{L}} \cdot \log\left(0.27 \cdot \frac{e}{D} + 4.62 \cdot \left(\frac{\nu D}{Q}\right)^{0.9}\right) \quad \textbf{[Eq. 163]}$$

This equation's variables are best solved for, by using a numerical spreadsheet application. Typically, there are three types of problems involving pipe friction losses, namely:

1. Head loss problem: calculate h_f given D, Q or V, and g, L, e, ν.
2. Discharge problem: calculate Q or V, given D, h_f and g, L, e, ν.
3. Sizing problem: calculate D, given Q, h_f and g, L, e, ν.

Where h_f is the friction loss,
D is the pipe diameter,
Q is the discharge,
V is the velocity,
g is the acceleration of gravity,
L is the pipe length,
e is the absolute roughness,
and, v is the kinematic viscosity.

Example 58 – Pipe flow solutions with the Darcy-Weisbach equation

Problem type 1. Given $D = 0.3\ ft$, $Q = 0.20\ cfs$, $g = 32.2\ ft/s^2$, $L = 1000\ ft$, $e = 0.002\ in = 0.000166\ ft$, and $\nu = 1.13x10^{-5}\ ft^2/s$, find h_f. A numerical spreadsheet application of equation 163 gives $h_f = 8.86\ ft$. Alternately, equation 163 can be solved directly for h_f, with a hand calculator.

Problem type 2. Given $D = 0.7\ ft$, $h_f = 15\ ft$, $g = 32.2\ ft/s^2$, $L = 750\ ft$, $e = 0.005\ in = 0.000416\ ft$, and $\nu = 1.2x10^{-5}\ ft^2/s$, find Q. A numerical spreadsheet application of equation 163 gives $Q = 2.68\ cfs$. Alternately, equation 163 can be solved iteratively for Q, with a hand calculator, as shown in the following table:

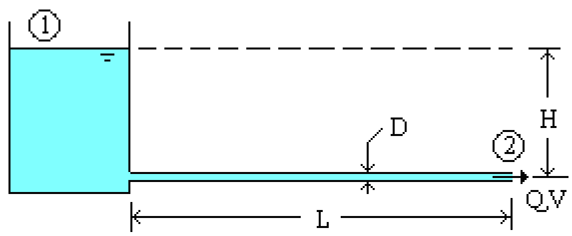
Trial Q	Calculated Q
1.5	2.63
2.1	2.66

2.6	2.681
2.7	2.684
2.68	2.683

Problem type 3. Given Q = 3 cfs, h_f = 10 ft, g = 32.2 ft/s^2, L = 1500 ft, e = 0.01 in = 0.000833 ft, and ν = 1.5x10^{-5} ft^2/s, find D. A numerical spreadsheet application of equation 163 gives D = 0.8591 ft × 12 in = 10.29 in. The recommended pipe diameter is 10.5 in.

Example 59 – Flow in pipeline draining a reservoir – the Darcy-Weisbach equation

Consider again (see example 56) a reservoir whose free surface is located at an elevation $z_1 = 60$ *ft*, draining through a *0.5-ft*-diameter, *100-ft*-long, concrete pipe (*e = 0.003 ft*) open to the atmosphere whose outlet is located at an elevation $z_2 = 55$ *ft*. The system, which carries water at a temperature of 55°F, is depicted in the following figure. Minor losses at the entrance from the reservoir into the pipe are ignored. Determine the discharge and velocity in the pipeline.



As in example 56, the energy equation (equation 47) is applied, but the friction losses are estimated with the Darcy-Weisbach equation instead of Manning's equation. The energy equation, in terms of discharge, *Q*, simplifies to:

$$H = \frac{8Q^2}{\pi^2 g D^4}\left(1 + f \cdot \frac{L}{D}\right)$$

In this equation, the friction factor, *f*, may be closely approximated by the Swamee-Jain equation (equation 159). Using *H = 5 ft*, *L = 100 ft*, *e = 0.003 ft*, $\nu = 1.3135 \times 10^{-5}$ *ft^2/s*, *D = 0.5 ft*, *g = 32.2 ft/s^2*, and applying a numerical spreadsheet solution gives *Q =1.285 cfs*.

And the flow velocity is:

$$V = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4 \times 1.285}{3.1416 \times 0.5^2} = 6.54\ ft/s.$$

0331.5 Pipe Flow Solutions Using Hazen-Williams Formula

A third method for calculating friction head losses in pipes is the Hazen-Williams formula. The Hazen-Williams formula was developed from empirical data of water flow in pipes. In terms of flow velocity, *V*, the Hazen-William formula is expressed as follows:

- International System (S.I.): *V*(m/s)

$$V = 0.849 \cdot C_{HW} \cdot R^{0.63} \cdot S_f^{0.54} \quad \textbf{[Eq. 164]}$$

- English System (E.S.): *V*(ft/s, fps)

$$V = 1.318 \cdot C_{HW} \cdot R^{0.63} \cdot S_f^{0.54} \quad \textbf{[Eq. 165]}$$

where *V* is velocity, C_{HW} is the Hazen-Williams coefficient, *R* is the hydraulic radius ($R=D/4$), and S_f is the energy slope ($S_f=h_f/L$). Using the definitions of *R* and S_f, the Hazen-Williams formula may be written as:

- International System (S.I.): h_f(m), *L*(m), *D*(m), *V*(m/s)

$$V = 0.354 \cdot C_{HW} \cdot D^{0.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} \quad \textbf{[Eq. 166]}$$

- English System (E.S.): h_f(ft), *L*(ft), *D*(ft), *V*(ft/s, fps)

$$V = 0.550 \cdot C_{HW} \cdot D^{0.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} \quad \textbf{[Eq. 167]}$$

In terms of the discharge, the Hazen-Williams formula is written as:

- International System (S.I.): h_f(m), *L*(m), *D*(m), *Q*(m^3/s)

$$Q = 0.278 \cdot C_{HW} \cdot D^{2.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} \quad \textbf{[Eq. 168]}$$

- English System (E.S.): h_f(ft), *L*(ft), *D*(ft), *Q*(cfs)

$$Q = 0.432 \cdot C_{HW} \cdot D^{2.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} \quad \textbf{[Eq. 169]}$$

Solving for the head loss, h_f, in the *Q*-based equations:

- International System (S.I.): h_f(m), *L*(m), *D*(m), *Q*(m^3/s)

$$h_f = 10.704 \cdot \left(\frac{Q}{C_{HW}} \right)^{1.85} \cdot \frac{L}{D^{4.87}}$$ **[Eq. 170]**

- English System (E.S.): h_f(ft), L(ft), D(ft), Q(cfs)

$$h_f = 4.732 \cdot \left(\frac{Q}{C_{HW}} \right)^{1.85} \cdot \frac{L}{D^{4.87}}$$ **[Eq. 171]**

Solving for the diameter, D, in the Q-based equations:

- International System (S.I.): h_f(m), L(m), D(m), Q(m^3/s)

$$D = 1.627 \cdot \left(\frac{Q}{C_{HW}} \right)^{0.38} \cdot \left(\frac{L}{h_f} \right)^{0.205}$$ **[Eq. 172]**

- English System (E.S.): h_f(ft), L(ft), D(ft), Q(cfs)

$$D = 1.376 \cdot \left(\frac{Q}{C_{HW}} \right)^{0.38} \cdot \left(\frac{L}{h_f} \right)^{0.205}$$ **[Eq. 173]**

Table 7, below, shows typical values for the Hazen-Williams coefficient. Hazen-Williams coefficients may be found in Lamont (1981), Mays (1991) and other similar publications.

Table 7. Values of the Hazen-Williams coefficient.

Pipe description	***Condition***	C_{HW}
Very smooth	Straight alignment	140
	Slight curvature	130
Cast iron, uncoated or steel pipe	New	130
	5 years old	120
	10 years old	110
	15 years old	100
	20 years old	90
	30 years old	80

Cast iron, coated	All ages	130
Wrought iron or standard galvanized steel	Diameter 12 in. and up	110
	Diameter 4 to 12 in.	100
	Diameter 4 in or less	80
Brass or lead	New	140
Concrete	Very smooth, excellent joints	140
	Smooth, good joints	120
	Rough	110
Vitrified clays		110
Smooth wooden; wood stave		120
Asbestos cement		140
Corrugated metal		60
Pipes of small diameter	old, rough inside surface, as low as	40
PVC		150
Smooth interior PE		150
Aluminum		120
Aluminum gated pipe		110

Solutions to pipe-flow problems with the Hazen-Williams formula require the use of equations (equation 164) through (equation 173), depending on the data given and the required variable to be solved for. Some examples are shown next.

Example 60 – Pipe velocity calculation using the Hazen-Williams formula

A pipe line with a length $L = 1200\ ft$, diameter $D = 0.75\ ft$, suffers a head loss $h_f = 12\ ft$. If the pipe is made of 5-year old steel pipe ($C_{HW} = 120$), determine the pipe velocity. Using equation 167 as follows:

$$V = 0.550 \cdot C_{HW} \cdot D^{0.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} = 0.550 \times 120 \times 0.75^{0.63} \cdot \left(\frac{12}{1200}\right)^{0.54} = 4.58\ fps$$

Example 61 – Pipe discharge calculation using the Hazen-Williams formula

Determine the discharge Q for a rough concrete pipeline ($C_{HW} = 110$) with a diameter $D = 1.5\ ft$, that produces a head loss $h_f = 8.5\ ft$ on a length of $L = 650\ ft$.
Using equation 169 as follows:

$$Q = 0.432 \cdot C_{HW} \cdot D^{2.63} \cdot \left(\frac{h_f}{L}\right)^{0.54} = 0.432 \times 110 \times 1.5^{2.63} \cdot \left(\frac{8.5}{650}\right)^{0.54} = 13.27\ cfs$$

Example 62 – Head loss calculation using the Hazen-Williams formula

Determine the head loss h_f in a new brass pipeline *(C_{HW} = 140)* of length *L = 2000 ft* and diameter *D = 3.0 ft*, that carries a discharge *Q = 20 cfs*.
Using equation 171 as follows:

$$h_f = 4.732 \cdot \left(\frac{Q}{C_{HW}}\right)^{1.85} \cdot \frac{L}{D^{4.87}} = 4.732 \cdot \left(\frac{20}{140}\right)^{1.85} \cdot \frac{2000}{3^{4.87}} = 1.23\ ft$$

Example 63 – Diameter calculation using the Hazen-Williams formula

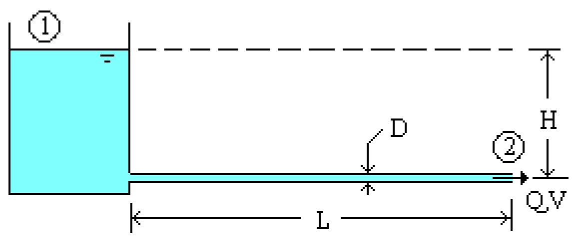
Determine the diameter of coated cast iron (C_{HW} *= 130*) pipe that produces a friction head loss h_f *= 20 ft* in a length *L = 500 ft* while carrying a discharge *Q = 25 cfs*.
Using equation 173 as follows:

$$D = 1.376 \cdot \left(\frac{Q}{C_{HW}}\right)^{0.38} \cdot \left(\frac{L}{h_f}\right)^{0.205} = 1.376 \cdot \left(\frac{25}{130}\right)^{0.38} \cdot \left(\frac{500}{20}\right)^{0.205} = 1.42\ ft$$

The most likely value of commercial pipeline that can be used is *D = 1.5 ft*.

Example 64 – Flow in pipeline draining a reservoir – Hazen-Williams formula

Consider once more (see examples 56 and 59) the case of a reservoir whose free surface is located at an elevation z_1 *= 60 ft*, draining through a *0.5-ft*-diameter, *100-ft*-long, rough concrete pipe (C_{HW} *= 110*) open to the atmosphere whose outlet is located at an elevation z_2 *= 55 ft*. The system is depicted in the following figure. Local losses at the entrance from the reservoir into the pipe are ignored. Determine the discharge and velocity in the pipeline.



As in examples 56 and 59, the energy equation (equation 47) is applied, but the friction losses are estimated with the Hazen-Williams formula. The energy equation, in terms of discharge, *Q*, simplifies to:

$$H = \frac{8Q^2}{\pi^2 g D^4} + 4.732 \cdot \left(\frac{Q}{C_{HW}}\right)^{1.85} \cdot \frac{L}{D^{4.87}}\ .$$

Using *H = 5 ft, L = 100 ft,* C_{HW} *= 110, D = 0.5 ft, g = 32.2 ft/s²*, and applying a numerical spreadsheet solution gives *Q = 1.385 cfs*, and the flow velocity is:

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 1.385}{3.1416 \times 0.5^2} = 7.05\ ft/s.$$

The same concrete pipe material, diameter, length, and elevations were used in examples 56, 59, and 64. These examples illustrate the use of Manning's equation, the Darcy-Weisbach equation (using Swamee-Jain equation for the friction factor), and the Hazen-Williams formula in estimating friction loss in pipe. The values of the flow velocity found using these three methods are listed below.

- Manning's equation: *V = 6.46 fps*
- Darcy-Weisbach (with Swamee-Jain): *V = 6.54 fps*
- Hazen-Williams formula: *V = 7.05 fps*

0331.6 Local Losses in Pipelines

Local losses are energy losses due to the presence of appurtenances or changes in the pipeline such as valves, elbows, curves, reductions or expansions. The term *local losses* indicate that these energy losses are concentrated at a location (rather than distributed along a pipeline as are friction losses). Local losses in pipelines are calculated using an equation of the form:

$$h_L = K \cdot \frac{V^2}{2g} = K \cdot \frac{8Q^2}{\pi\, gD^4} \qquad \textbf{[Eq. 174]}$$

Where the *local loss coefficient K*, depends on the nature of the device or pipeline change producing the loss. Note that local losses are sometimes referred to as minor losses.

In addition to losses at appurtenances, local losses occur at pipeline expansions and contractions; see section 0332.1. Also, local losses occur at entrances and discharge ends of pipe.

<u>Discharge loss</u>

Figure 51, below, shows the conditions of flow at the discharge end of a pipeline into a reservoir.

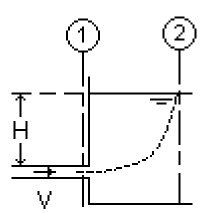


Figure 51. Discharge from a pipe into a reservoir.

At section (1), right before the entrance, the pressure head is $p_1/\omega = H$, the elevation of the pipe centerline can be taken as $z_1 = 0$, and the velocity is $V_1 = V$. The curved line in the reservoir depicts a streamline of flow connecting the pipe outlet to the free surface of the reservoir at section (2). In this section the pressure head is $p_2/\omega = 0$, the elevation is $z_2 = H$, and the local velocity is $V_2 = 0$. Energy head loss between sections (1) and (2) is a local head loss referred to as a *discharge loss*, and given by equation 174:

$$(h_L)_d = K_d \cdot \frac{V^2}{2g}$$

Writing the energy equation (equation 47) between points (1) and (2), gives:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_L$$

Replacing the data for sections (1) and (2) as described above, the equation reduces to:

$$0 + H + \frac{V^2}{2g} = H + 0 + 0 + K_d \cdot \frac{V^2}{2g}$$

From which it follows that the discharge loss coefficient, $K_d = 1.0$ (standard value used in pipe flow analyses).

<u>Entrance loss</u>
Three different conditions of entrance from a reservoir into a pipe, and their local loss coefficients, are depicted below.

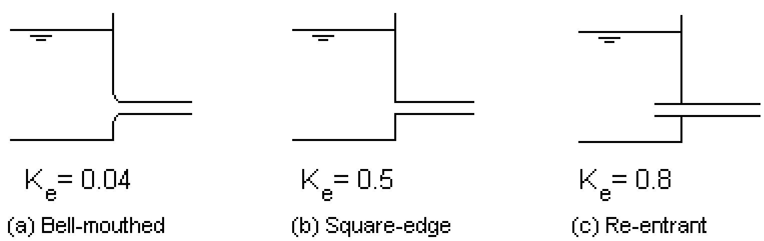


Figure 52. Entrance loss coefficients for typical pipe entrance shapes

The *Pipe Flow* tab in the *USDA-NRCS Hydraulics Formula* software provides additional values for the entrance coefficient K_e for other specific entrance conditions. These values are summarized in Table 8, below. Entrance loss coefficients may also be found in FHWA (2001) and USACOE (2008), related to culvert analysis.

Entrance losses are calculated using the expression (equation 174):

$$(h_L)_e = K_e \cdot \frac{V^2}{2g} = K_e \cdot \frac{8Q^2}{\pi^2 g D^4}$$

Table 8. Pipe entrance loss coefficients.

Type of Structure and Entrance Design	K_e
Pipe, Concrete	
Projecting from fill, socket end [groove-end]	0.2
Projecting from fill, square cut end	0.5
Headwall or Headwall and wingwalls	
Socket end of pipe [groove-end]	0.2
Square-end	0.5
Rounded (radius = 1/12D)	0.2
Mitered to conform to fill slope	0.7
End section conforming to fill slope	0.5
Pipe or Pipe-Arch, Corrugated Metal	
Projecting from fill (no headwall)	0.9
Headwall or Headwall and wingwalls square-edge	0.5
Mitered to conform to fill slope	0.7
End section conforming to fill slope	0.5

Example 65 – Pipe flow calculation including local losses using the *USDA-NRCS Hydraulics Formula* software

Using the *Pipe Flow* tab in the *USDA-NRCS Hydraulics Formula* software, calculate the discharge Q and flow velocity V for a *18-in*-diameter, *1200-ft*-long, corrugated plastic pipe (Manning's $n = 0.015$) whose entrance is mitered to conform to a fill slope ($K_e = 0.7$). Head on the pipe is *10 ft*.

The solution, illustrated in the figure below indicates that $Q = 8.1$ *cfs*, and $V = 4.6$ *ft/s*.

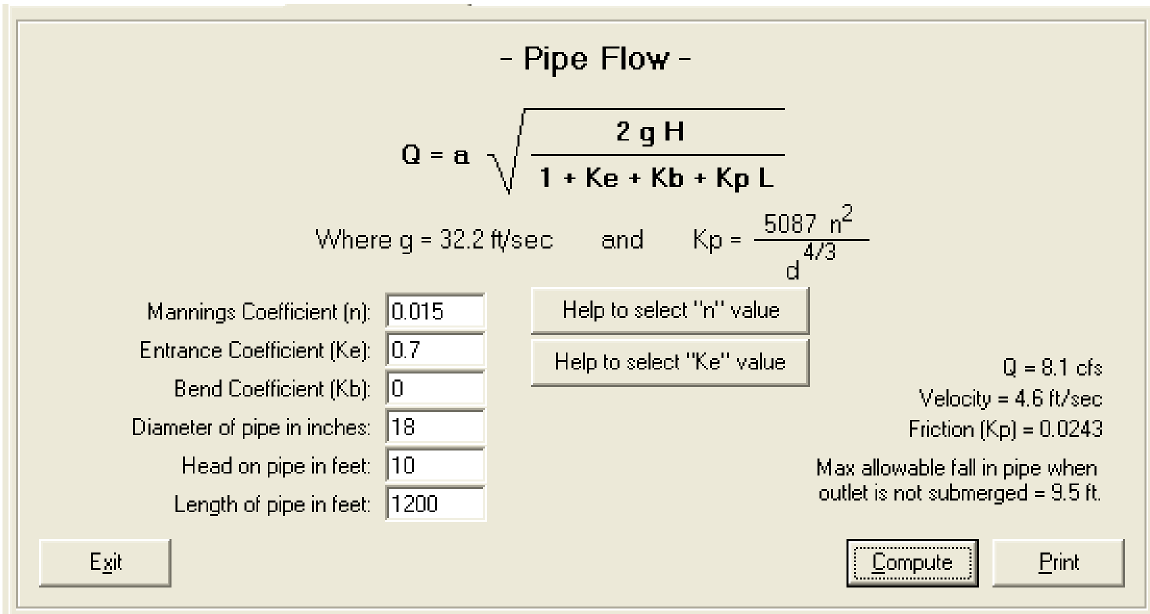


Figure 53. Solution for pipe flow with entrance loss using *USDA-NRCS Hydraulics Formula*.

Losses due to pipe fittings

Pipe fittings such as valves, elbows, and bends produce local losses according to (equation 174). The values of selected pipe fittings are shown in Table 9, below.

Table 9. Local loss coefficients for selected pipe fittings.

Pipe fitting	*K*
Globe valve, wide open	10
Alfalfa or stub valves	2.80
Close-return bend	2.20
Portable hydrants	1.00
Valve opening elbows	1.00
T, through side outlet	1.80
short-radius elbow	0.90
medium-radius elbow	0.75
long-radius elbow	0.60
45^o elbow	0.42
Gate valve, wide open	0.19
Gate valve, half open	2.06

For a more complete set of values for local losses, refer to Brater and King (1996) or Idelchik (1999).

Example 66 – Pipe flow between two reservoirs including local losses – Manning's equation

The figure below shows the steady flow between two reservoirs whose free surfaces have an elevation difference, *H*. The pipe has length *L*, diameter *D*, and resistance coefficient; the pipe carries a discharge *Q* and flows with a velocity, *V*.

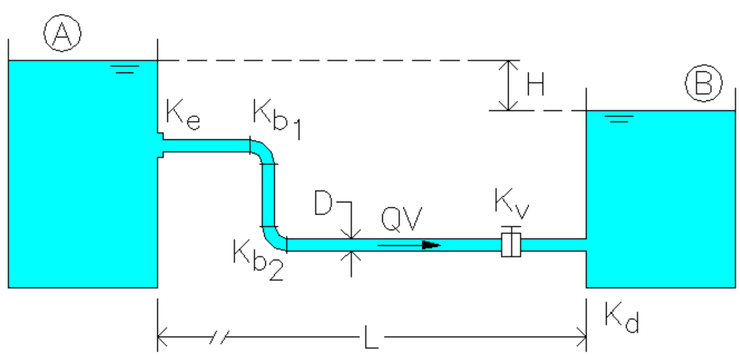


Figure 54. Schematic of pipe flow between two reservoirs.

The velocity at the free surfaces (A) and (B) is practically zero, i.e., $V_A = V_B = 0$, and the gage pressure at those points is also zero, $p_A = p_B = 0$. Finally, the elevations of the free surfaces at points (A) and (B) can be taken as $z_A = H$ and $z_B = 0$ (i.e., the reference level for elevation is free surface (B)). The energy equation between points (A) and (B), including friction and local losses in the pipe, is written as (see equation 51):

$$z_A + \frac{p_A}{\omega} + \frac{V_A^2}{2g} = z_B + \frac{p_B}{\omega} + \frac{V_B^2}{2g} + h_f + \sum h_L$$

Introducing Manning's equation (equation 151) to estimate pipe friction losses, the local loss equation (equation 174), and the data presented above, the energy equation is written as:

$$z_B + H + \frac{0}{\omega} + \frac{0}{2g} = z_B + \frac{0}{\omega} + \frac{0}{2g} + 2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} \cdot V^2 + \sum K \cdot \frac{V^2}{2g}$$

Results in: $$H = V^2 \cdot \left(2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} + \frac{\sum K}{2g} \right)$$

In terms of the flow discharge, this equation is written as:

$$H = \frac{16 \cdot Q^2}{\pi^2 D^4} \cdot \left(2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} + \frac{\sum K}{2g} \right)$$

The term ΣK includes the sum of all local loss coefficients in the pipeline. Note that the Darcy-Weisbach equation or the Hazen-Williams formula is always an option to use in estimating pipe friction loss.

Using the values L = *550 ft*, *D = 2 ft*, *n = 0.012*, *Q = 35 cfs*, with the following local loss coefficients:

- Entrance: $K_e = 0.04$ (bell-mouth entrance)
- Elbow 1: $K_{b1} = 0.90$ (short-radius elbow)
- Elbow 2: $K_{b2} = 0.75$ (medium-radius elbow)
- Valve: $K_v = 0.19$ (gate valve, wide open)
- Discharge: $K_d = 1.00$ (standard value for discharge coefficient)

Thus, for this case $\Sigma K = K_e + K_{b1} + K_{b2} + K_v + K_d = 0.04+0.90+0.75+0.19+1.00 = 2.88$. Find the difference in elevations between the reservoirs, *H*.

Using the equation developed above:

$$H = \frac{16 \times 35^2}{3.1416^2 \times 2^4} \cdot \left(2.8755 \times \frac{0.012^2 \times 550}{2^{4/3}} + \frac{2.88}{2 \times 32.2} \right)$$

$$H = 16.77 \text{ ft}.$$

If the elevation difference between reservoirs is known, the resulting equation above may be solved for V to determine the velocity in the pipe. Then the discharge may be determined with the continuity eqution, $Q = VA$.

<u>Development of the *Pipe Flow* equation</u>

The equation from the *Pipe Flow* tab in the *USDA-NRCS Hydraulics Formula* software is shown Figure 53 and repeated here:

$$Q = a \cdot \sqrt{\frac{2gH}{1+K_e+K_b+K_pL}}, \quad Kp = \frac{5087 \cdot n^2}{d^{4/3}}, \quad d(in) \qquad \textbf{[Eq. 175]}$$

In order to develop this equation:

$$\Sigma K = K_e + K_b + 1$$

with K_e being the entrance loss coefficient, K_b accounting for devices such as elbows and valves (referred to as *Bend Coefficient* in Figure 53), and $K_d = 1$ being the discharge coefficient. Thus, the energy equation presented in example 66 (Manning's equation is used to estimate pipe friction losses) is written as:

$$H = V^2 \cdot \left(2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} + \frac{\sum K}{2g}\right) = \frac{V^2}{2g} \cdot \left(2g \times 2.8755 \cdot \frac{n^2 \cdot L}{D^{4/3}} + K_e + K_b + 1\right)$$

Replacing the diameter D (in *feet*) with d (in *inches*) and defining:

$$K_p = \frac{5087 \cdot n^2}{d^{4/3}}$$

The energy equation becomes:

$$H = \frac{V^2}{2g} \cdot \left(K_p \cdot L + K_e + K_b + 1\right) = \frac{V^2}{2g} \cdot \left(1 + K_e + K_b + K_p \cdot L\right)$$

Solving for V, gives the following equation:

$$V = \sqrt{\frac{2gH}{1+K_e+K_b+K_p \cdot L}}$$

Multiplying this result by the area of the cross-section and substituting Q for VxA gives equation 175.

Example 67 – Pipe flow calculation including local losses using *USDA-NRCS Hydraulics Formula*

Use the *Pipe Flow* tab in the *USDA-NRCS Hydraulics Formula* software to determine the discharge in a pipe with $L = 550\ ft$, $D = 2\ ft$, $n = 0.012$, $H = 10\ ft$, with the following local loss coefficients:

- Entrance: $K_e = 0.04$ (bell-mouth entrance)
- Bend coefficient: $K_{b2} = 0.75$ (medium-radius elbow)
- Discharge: $K_d = 1.00$ (standard value for discharge coefficient)

The solution is shown in the following Figure 55. The results shown are $Q = 28.9\ cfs$ and $V = 9.2\ fps$. Also, the friction coefficient $K_p = 0.0106$.

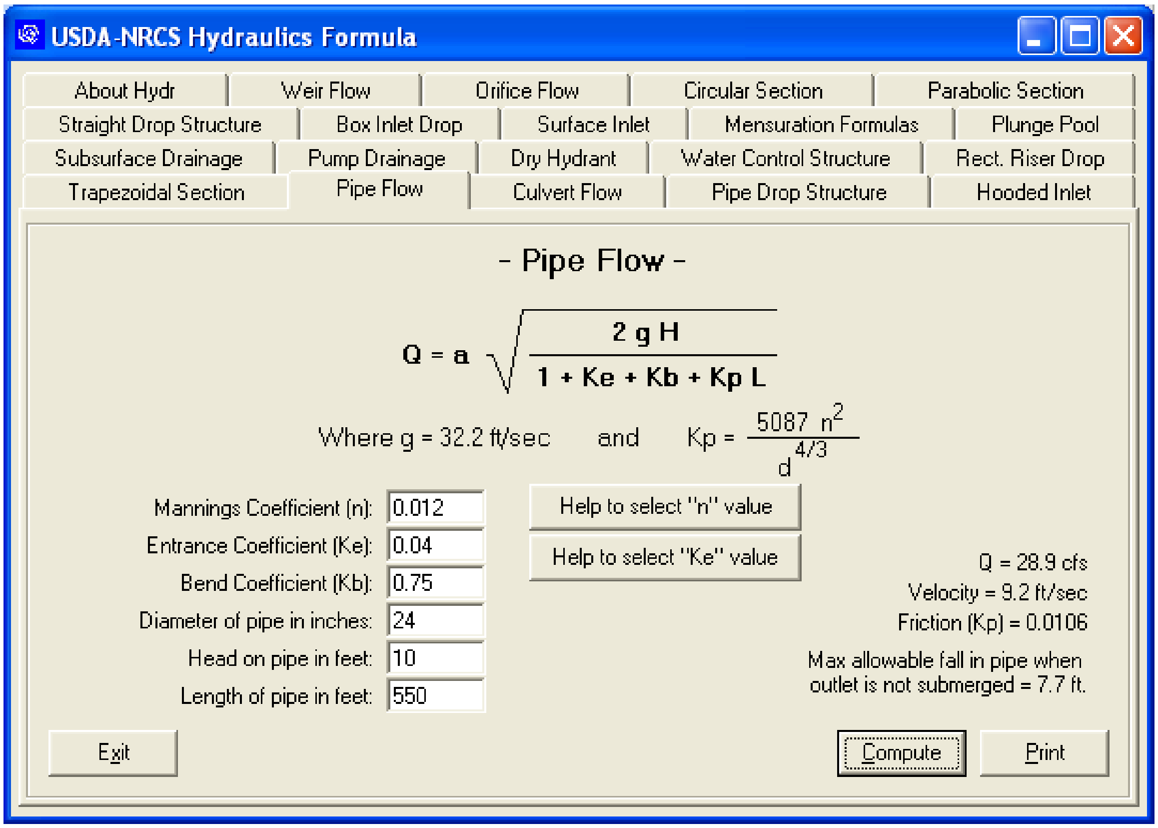


Figure 55. Solution for pipe flow including all local lossess using *USDA-NRCS Hydraulics Formula*

0331.7 Pumps in Pipelines

Pumps are mechanical devices used to introduce energy into a pipeline system. Pumps can be used, for example, to lift water from a lower elevation to a higher one, or to overcome friction losses between reservoirs. The most common types of pumps used in pipelines are centrifugal pumps. In this section the analysis of pipeline systems with centrifugal pumps is presented.

A pump introduces an energy head h_p into a pipeline system. The energy equation including a pump head is written as:

$$z_1 + \frac{p_1}{\omega} + \frac{V_1^2}{2g} + h_p = z_2 + \frac{p_2}{\omega} + \frac{V_2^2}{2g} + h_f + \sum h_L \qquad \textbf{[Eq. 176]}$$

The normal convention is that energy added to the flow, such as the pump head h_p, is placed on the left-hand side (upstream side) of the equation, while, energy extracted from or lost by the flow, such as the friction head h_f and the sum of local losses Σh_L, is placed on the right-hand side (downstream side) of the equation.

0331.7.1 Pump Operational Characteristics

In pipeline system hydraulics, an important operational characteristic of a pump is the variation of the pump head h_p with the discharge Q passing through the pump. For centrifugal pumps the relationship between h_p and Q is given by a quadratic equation (also called a polynomial equation of the second degree) of the form:

$$h_p = aQ^2 + bQ + c \qquad \textbf{[Eq. 177]}$$

The coefficients a, b, and c, in this equation, can be determined by fitting data from tests performed on a given pump. The equation describing a given pump, or at least a graph of the relationship between h_p and Q, should be available from the pump manufacturer.

Example 68 – Pump head and discharge analysis

Tests on a pump produce the following discharge-head data:

Q(cfs)	*h_p(ft)*
0.0	25.3
5.0	22.0
10.0	18.3
15.0	12.0
20.0	4.2

The pump discharge graph and equation were produced with curve-fitting software and are shown in Figure 56 below.

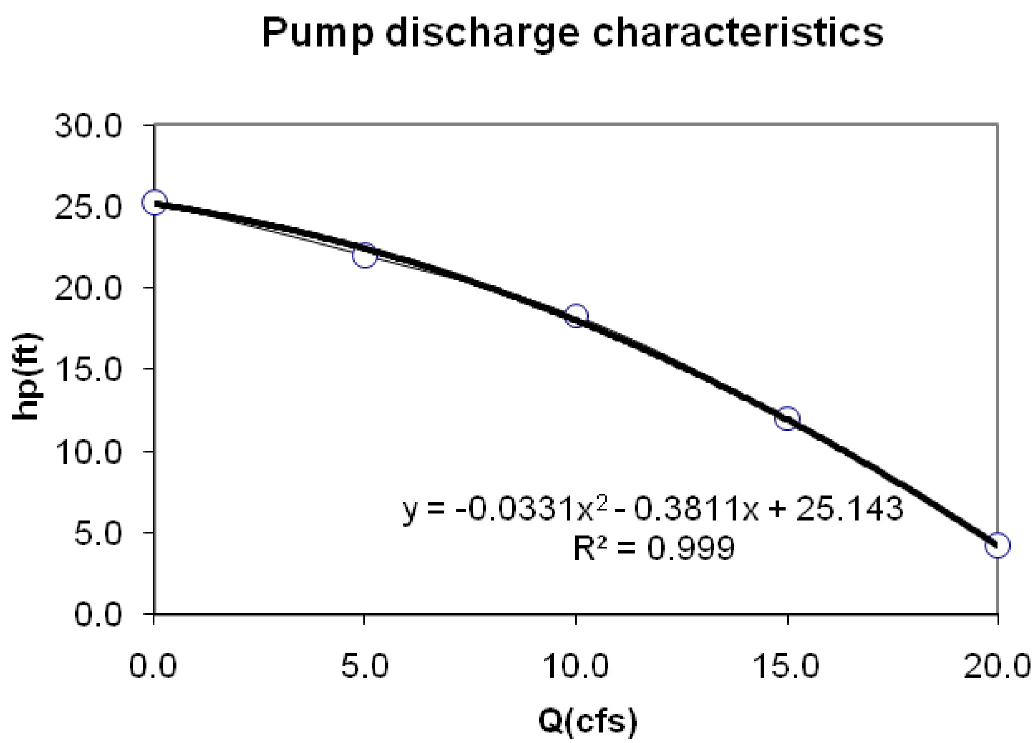


Figure 56. Pump discharge-head graph

Notice from the graph that the energy head for zero discharge is $h_p = 25.143$ *ft*. This is the "shut-off head" or maximum head for this pump. If valves are closed and no flow occurs, the pump is capable of building *25.143 ft* of head.

The maximum pump capacity at free discharge is calculated from the pump curve equation by setting $h_p\ (y) = 0$ and solving for Q. This gives $0 = -0.0331Q^2 - 0.3811Q + 25.143$. The positive solution to this quadratic equation produces the free-discharge, $Q = 22.4$ *cfs*, which the pump delivers with $h_p = 0$ (no discharge pressure). This discharge is what the pump would deliver if it were disconnected from any pipeline and allowed to discharge freely to the atmosphere.

An example of a pump-pipeline system analysis using manufacturer-provided pump curves is presented in

Exhibit 6 - Pipe-system analysis .

0331.7.2 Pump Power and Efficiency

The hydraulic power developed by a pump represents the amount of energy per unit time introduced by the pump into the flow. The hydraulic power, P_h, is calculated as:

$$P_h = \omega Q h_p \qquad \textbf{[Eq. 178]}$$

where ω is the specific weight of water. Using the units of the English system, namely, ω (lb/ft^3), Q (cfs), and h_p(ft), the power is given in units of *lb·ft/s*. A more commonly used unit of power in the English System is the *horsepower* (*hp*) defined as *1 hp = 550 lb·ft/s*. Thus, in terms of horsepower the equation for the hydraulic power of a pump is:

$$P_h = \frac{\omega Q h_p}{550},\ P_h(hp) \qquad \textbf{[Eq. 179]}$$

In units of the International System (S.I.), namely, ω(N/m^3), Q (m^3/s), and h_p(m), the pump power P_h is given in *Watts* (*W*). For the S,I, use equation 178 to calculate the power.

To provide hydraulic power, P_h, the pump ***P*** is activated by a motor ***M*** as illustrated in Figure 57, below. The motor can be powered by electricity at connection ***E***.

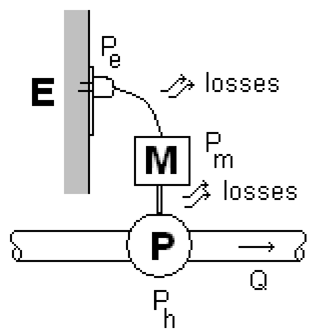


Figure 57. System schematic showing motor *M*, the electric supply *E*, and pump *P*

The motor provides the pump with input power P_m, however, due to energy losses by friction in the pump shaft (which is dissipated as heat in the environment), and other losses, not all of the motor power P_m is utilized by the pump to produce hydraulic power. Thus, in general, $P_h < P_m$, and the ratio between the hydraulic power and the motor power is a quantity smaller than one ($P_h/P_m<1$) known as the *efficiency of the pump,* η_p:

$$\eta_p = \frac{P_h}{P_m} < 1 \qquad \textbf{[Eq. 180]}$$

The motor itself is provided with a certain amount of electric power P_e; however, due to losses in the transmission line as well as in the motor mechanism the motor power P_m is smaller than the electric power, i.e., $P_m<P_e$. Thus, the *efficiency of the motor,* η_m, is a quantity smaller than one ($P_m/P_e <1$), defined as:

$$\eta_m = \frac{P_m}{P_e} < 1 \qquad \textbf{[Eq. 181]}$$

The combined motor-pump system, in turn, has an efficiency $\eta_{m\text{-}p}$ defined as

$$\eta_{m-p} = \frac{P_h}{P_e} = \frac{P_h}{P_m} \cdot \frac{P_m}{P_e} = \eta_p \cdot \eta_m < 1 \qquad \textbf{[Eq. 182]}$$

The various efficiencies, η_i, can be expressed as a percent by multiplying by 100.

Example 69 - Pump power and efficiency calculation

A pump produces a head $h_p = 41.67\ ft$ with a discharge $Q = 1.24\ cfs$. If the pump efficiency is 80% ($\eta_p = 0.80$) and the motor efficiency is 90% ($\eta_m = 0.90$), determine: (a) the hydraulic power provided by the pump to the flow, P_h; (b) the power that the motor needs to provide to the pump, P_m; (c) the electric power needed to be provided to the motor, P_e; and (d) the efficiency of the motor-pump system, $\eta_{m\text{-}p}$.

(a) The hydraulic power (in *hp*) is calculated according to equation 179:

$$P_h = \frac{\omega Q h_p}{550} = \frac{62.4 \times 1.24 \times 41.67}{550} = 5.86\, hp$$

(b) The power provided by the motor follows from the definition of the pump efficiency (equation 180):

$$P_m = \frac{P_h}{\eta_p} = \frac{5.86}{0.80} = 7.33\, hp$$

(c) The electric power provided to the motor follows from the definition of the motor efficiency (equation 181):

$$P_e = \frac{P_m}{\eta_m} = \frac{7.33}{0.90} = 8.14\ hp$$

(d) The efficiency of the motor-pump system can be calculated with equation 182:

$$\eta_{m-p} = \eta_p \cdot \eta_m = 0.80 \times 0.90 = 0.72 = 72\%$$

or

$$\eta_{m-p} = \frac{P_h}{P_e} = \frac{5.86}{8.14} = 0.72 = 72\ \%$$

0332 Pipelines and Networks

Pipelines of different diameters can be combined into different configurations including pipes in series, pipes in parallel, pipes converging to a single point, and more complex pipe networks. These configurations are discussed in the following sections.

0332.1 Pipelines in Series

Pipelines in series consist of segments of pipeline connected one after the other so that the flow discharge follows a single path. An arrangement of pipelines in series is presented in Figure 58, below.

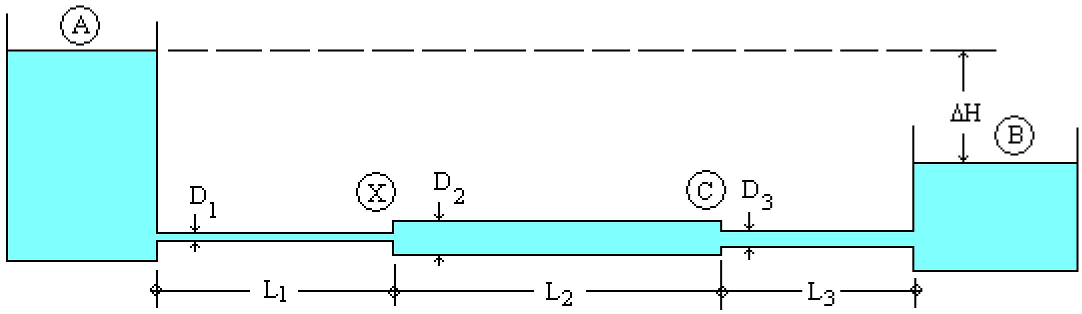


Figure 58. Three pipelines in series connecting two reservoirs.

Three pipelines of different diameters and lengths are shown carrying water between reservoirs (A) and (B). The pipelines have lengths L_1, L_2, and L_3, with corresponding diameters D_1, D_2, and D_3. In addition, the pipelines have different roughness coefficients. The elevation difference between reservoirs (A) and (B) is given by ΔH.

Local head losses due to an expansion or a contraction
The system illustrated in Figure 58 shows an expansion in diameter between pipelines 1 and 2 (point X), and a contraction in diameter between pipelines 2 and 3 (point C). Expansions and contraction in pipelines produce local head losses.

For a sudden expansion, the local head losses are calculated as:

$$(h_L)_x = \frac{(V_u - V_d)^2}{2g} = \left[1 - \left(\frac{D_u}{D_d}\right)^2\right]^2 \cdot \frac{V_u^{\ 2}}{2g} = \left[\left(\frac{D_d}{D_u}\right)^2 - 1\right]^2 \cdot \frac{V_d^{\ 2}}{2g} \qquad \textbf{[Eq. 183]}$$

The subscripts, *u* and *d* refer to the upstream and downstream pipelines. Notice that the head loss in a sudden expansion depends only on the diameters upstream and downstream of the expansion (D_u, D_d). Defining expansion loss coefficients as:

$$K_{xu} = \left[1 - \left(\frac{D_u}{D_d}\right)^2\right]^2 \qquad \textbf{[Eq. 184]}$$

or

$$K_{xd} = \left[\left(\frac{D_d}{D_u}\right)^2 - 1\right]^2 \qquad \textbf{[Eq. 185]}$$

The general form of the local losses equation is written as:

$$(h_L)_x = K_{xu} \cdot \frac{V_u^{\ 2}}{2g} = K_{xd} \cdot \frac{V_d^{\ 2}}{2g} \qquad \textbf{[Eq. 186]}$$

For a sudden contraction, as the one between pipes 2 and 3 in Figure 58, the local head losses are given by the equation:

$$(h_L)_c = K_c \cdot \frac{V_d^{\ 2}}{2g} \qquad \textbf{[Eq. 187]}$$

where K_c is the contraction loss coefficient given in Table 10, below, and V_d is the flow velocity in the downstream pipeline, i.e., the one with the smallest diameter. In Table 10, D_d and D_u are the diameters of the pipe downstream and upstream of the contraction. See Brater and King (1996) or Idelchik (1999) for more information on pipeline contraction losses.

Table 10. Local head loss coefficients for a sudden pipe contraction.

D_d/D_u	K_c	D_d/D_u	K_c
0.1	0.45	0.6	0.28
0.2	0.42	0.7	0.22
0.3	0.39	0.8	0.15
0.4	0.36	0.9	0.06
0.5	0.33	1.0	0.00

Energy equation for the three-pipeline system

In writing the energy equation between reservoirs (A) and (B) in Figure 58 friction head losses in each pipeline will be included, as well as, entrance losses from reservoir (A) into pipeline 1, expansion losses at point (X), contraction losses at point (C), and discharge losses from pipeline 3 into reservoir (B). For the points (A) and (B) in the surface of the reservoirs, $V_A = V_B = 0$, $p_A = p_B = 0$, $z_A = z_B + \Delta H$. The expressions for the local losses are the following:

- Entrance loss from reservoir (A) to pipe 1: $(h_L)_e = K_e \cdot \frac{V_1^2}{2g}$, with $K_e = 0.5$. See Figure 52.

- Expansion from pipe 1 to pipe 2: $(h_L)_x = K_{x1} \cdot \frac{V_1^2}{2g}$, with $K_{x1} = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2$
- Contraction from pipe 2 to pipe 3: $(h_L)_c = K_c \cdot \frac{V_3^2}{2g}$, with K_c from Table 10 as a function of the ratio D_3/D_2.
- Discharge loss from pipe 3 into reservoir (B): $(h_L)_d = K_d \cdot \frac{V_3^2}{2g}$, with $K_d = 1.0$, always.

The friction losses in each pipe may be calculated using either the Darcy-Weisbach equation, the Manning's equation, or the Hazen-Williams formula. The total friction loss is the sum of each pipe's friction loss.

The energy equation for the system of Figure 58 is written as:

$$z_A + \frac{p_A}{\omega} + \frac{V_A^2}{2g} = z_B + \frac{p_B}{\omega} + \frac{V_B^2}{2g} + \sum h_f + \sum h_L$$

From which it follows that:

$$z_B + \Delta H + \frac{0}{\omega} + \frac{0}{2g} = z_B + \frac{0}{\omega} + \frac{0}{2g} + \sum h_f + K_e \cdot \frac{V_1^2}{2g} + K_{x1} \cdot \frac{V_1^2}{2g} + K_c \cdot \frac{V_3^2}{2g} + K_d \cdot \frac{V_3^2}{2g}$$

or,

$$\Delta H = \sum h_f + \sum h_L = \sum h_f + K_e \cdot \frac{V_1^2}{2g} + K_{x1} \cdot \frac{V_1^2}{2g} + K_c \cdot \frac{V_3^2}{2g} + K_d \cdot \frac{V_3^2}{2g} \quad \textbf{[Eq. 188]}$$

In most analyses of pipelines in series the diameters of the pipelines are known, and the problem consists in determining the discharge Q for a given available head ΔH, or vice versa. The three examples of pipes in series presented below include the effect of local losses.

Example 70 – Pipes in series using the Darcy-Weisbach equation

The system of Figure 58 has pipelines lengths, diameters, and roughness values: $L_1 = 200$ *ft*, $L_2 = 400$ *ft*, $L_3 = 150$ *ft*, $D_1 = 1.00$ *ft*, $D_2 = 1.50$ *ft*, $D_3 = 1.00$ *ft*, $e_1 = 0.0001$ *ft(concrete)*, $e_2 = 0.00004$ *ft(PVC)*, and $e_3 = 0.00025$ *ft(welded steel)*. The kinematic viscosity is $\nu = 1\times10^{-5}$ ft^2/s. (a) If the discharge $Q = 5$ *cfs*, determine the needed head ΔH. (b) If the available head between the reservoirs is $\Delta H = 30$ *ft*, determine the discharge through the pipes.

A spreadsheet application gives the results. If the discharge is *5 cfs*, the needed head is coincidentally *5 ft*. If the available head is *30 ft*, the discharge, $Q = 12.58$ *cfs*.

Example 71 - Pipes in series using the Manning's equation

Using the same data as example 70, the pipe lengths, diameters, and Manning's *n* coefficients are given by $L_1 = 200$ *ft*, $L_2 = 400$ *ft*, $L_3 = 150$ *ft*, $D_1 = 1.00$ *ft*, $D_2 = 1.50$ *ft*, $D_3 = 1.00$ *ft*, $n_1 = 0.012$*(concrete)*, $n_2 = 0.01$*(PVC)*, and $n_3 = 0.013$*(welded steel)*. (a) If the discharge $Q = 5$ *cfs*, determine the needed head ΔH. (b) If the available head between the reservoirs is $\Delta H = 30$ *ft*, determine the discharge through the pipes.

A spreadsheet application gives the results. If the discharge is *5 cfs*, the needed head is *8.12 ft*. If the available head is *30 ft*, the discharge, $Q = 9.61$ *cfs*.

Example 72 - Pipes in series using the Hazen-Williams formula

Using the same data as examples 70 and 71, the pipe lengths, diameters, and Hazen-Williams coefficients are given by $L_1 = 200$ *ft*, $L_2 = 400$ *ft*, $L_3 = 150$ *ft*, $D_1 = 1.00$ *ft*, $D_2 = 1.50$ *ft*, $D_3 = 1.00$ *ft*, $C_{HW1} = 120$*(concrete)*, $C_{HW\,2} = 150$*(PVC)*, and $C_{HW\,3} = 120$*(welded steel)*. (a) If the discharge $Q = 5$ *cfs*, determine the needed head ΔH. (b) If the available head between the reservoirs is $\Delta H = 30$ *ft*, determine the discharge through the pipes.

A spreadsheet application gives the results. If the discharge is *5 cfs*, the needed head is *6.37 ft*. If the available head is *30 ft*, the discharge, $Q = 11.39$ *cfs*.

Calculations were also made ignoring local losses in examples 70, 71, and 72. Table 11 shows the effect of neglecting local losses on discharge.

Table 11. Effect of neglecting local losses on an example pipeline system

Friction loss method	*Q (cfs) including local losses*	*Q (cfs) neglecting local losses*	*Percentage difference*
Darcy-Weisbach	*12.58*	*14.79*	*17.56%*
Manning's	*9.61*	*10.47*	*8.95%*
Hazen-Williams	*11.39*	*13.03*	*14.40%*

The percentage differences in Table 11 are too large to neglect local loss. A generally accepted criterion is that local losses should be included in a pipe design analysis if the local losses exceed 5% of the total head loss.

0332.2 Pipelines in Parallel

Figure 59 illustrates reservoirs connected by three different parallel pipelines conducting water between the reservoirs. An energy equation can be written separately for each of the pipelines. Including entrance (h_e), discharge (exit) (h_d), and friction (h_f) losses for each pipeline, the corresponding energy equations can be written as:

$$\Delta H = (h_e)_1 + (h_d)_1 + (h_f)_1$$
$$\Delta H = (h_e)_2 + (h_d)_2 + (h_f)_2 \qquad \textbf{[Eqs. 189]}$$
$$\Delta H = (h_e)_3 + (h_d)_3 + (h_f)_3$$

Where the subscripts (1), (2), and (3) refer to each of the pipelines as illustrated in the figure below.

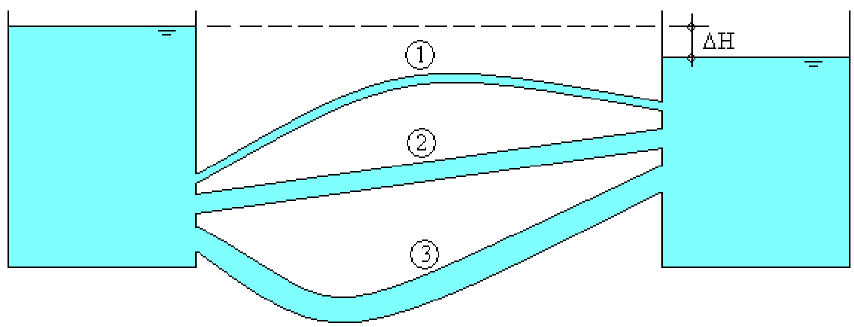


Figure 59. Three pipelines in parallel connecting two reservoirs.

A simplified analysis in which the local losses are neglected simplifies equation 189 to:

$$\Delta H = (h_f)_1 = (h_f)_2 = (h_f)_3 \qquad \textbf{[Eq. 190]}$$

A typical problem of parallel pipes consists in determining the total discharge Q delivered from the upstream to the downstream reservoir given the available head ΔH between the reservoirs. Given ΔH, the available head, the friction loss, h_f, is known. Then using either the Darcy-Weisbach equation (requires spreadsheet application for efficient calculation), the Manning's equation, or the Hazen-Williams formula, the individual discharges Q_1, Q_2, and Q_3 can be obtained. The individual discharges are summed to obtain the total discharge. The examples using Manning's equation and the Hazen-Williams formula presented below neglect local losses.

Example 73 - Pipes in parallel using the Manning's equation

The system of Figure 59 has pipelines lengths, diameters, and Manning's n coefficients: $L_1 = 200\ ft$, $L_2 = 400\ ft$, $L_3 = 150\ ft$, $D_1 = 1.00\ ft$, $D_2 = 1.50\ ft$, $D_3 = 1.00\ ft$, $n_1 = 0.012$, $n_2 = 0.018$, and $n_3 = 0.010$. If the available head is $\Delta H = 30\ ft$, determine the total discharge between the reservoirs.

The head loss through each pipeline is *30 ft*. Manning's equation (equation 152) is used to calculate the individual discharges:

$$Q_1 = 0.4632 \cdot \frac{D_1^{8/3}}{n_1} \cdot \sqrt{\frac{(h_f)_1}{L_1}} = 0.4632 \times \frac{1.00^{8/3}}{0.012} \times \sqrt{\frac{30}{200}} = 14.95\,cfs$$

$$Q_2 = 0.4632 \cdot \frac{D_2^{8/3}}{n_2} \cdot \sqrt{\frac{(h_f)_2}{L_2}} = 0.4632 \times \frac{1.50^{8/3}}{0.018} \times \sqrt{\frac{30}{400}} = 20.78\,cfs$$

$$Q_3 = 0.4632 \cdot \frac{D_3^{8/3}}{n_3} \cdot \sqrt{\frac{(h_f)_3}{L_3}} = 0.4632 \times \frac{1.00^{8/3}}{0.010} \times \sqrt{\frac{30}{150}} = 20.71\,cfs$$

The total discharge is $Q = 14.95\ cfs + 20.78\ cfs + 20.71\ cfs = 56.44\ cfs$.

Discharge calculations may also be made for individual pipes with *Pipe Flow, NRCS Hydraulics Formula* program, which uses Manning's equation for friction losses. The discharges calculated will be slightly less than those shown above because *Pipe Flow, NRCS Hydraulics Formula* automatically accounts for a local loss at the pipe exit ($K_d = 1.0$).

Example 74 - Pipes in parallel using the Hazen-Williams formula
The system of Figure 59 has pipelines lengths, diameters, and Hazen-Williams coefficients: $L_1 = 200\ ft$, $L_2 = 400\ ft$, $L_3 = 150\ ft$, $D_1 = 1.00\ ft$, $D_2 = 1.50\ ft$, $D_3 = 1.00\ ft$, $C_{HW1} = 100$, $C_{HW2} = 80$, and $C_{HW3} = 120$. If the available head is $\Delta H = 30\ ft$, determine the total discharge between the reservoirs.

The head loss through each pipeline is *30 ft*. Using (equation 169), the individual pipe discharges are calculated as follows:

$$Q_1 = 0.432 \cdot C_{HW1} \cdot D_1^{2.63} \cdot \left(\frac{(h_f)_1}{L_1}\right)^{0.54} = 0.432 \times 100 \times 1.00^{2.63} \times \left(\frac{30}{200}\right)^{0.54} = 15.50\,cfs$$

$$Q_2 = 0.432 \cdot C_{HW2} \cdot D_2^{2.63} \cdot \left(\frac{(h_f)_2}{L_2}\right)^{0.54} = 0.432 \times 80 \times 1.50^{2.63} \times \left(\frac{30}{400}\right)^{0.54} = 24.79\,cfs$$

$$Q_3 = 0.432 \cdot C_{HW3} \cdot D_3^{2.63} \cdot \left(\frac{(h_f)_3}{L_3}\right)^{0.54} = 0.432 \times 120 \times 1.00^{2.63} \times \left(\frac{30}{150}\right)^{0.54} = 21.74\,cfs$$

The total discharge is $Q = 15.50\ cfs + 24.79\ cfs + 21.74\ cfs = 62.03\ cfs$.

0332.3 Pipelines Converging at a Single Point

Figure 60 shows a pipeline system consisting of three pipelines connecting reservoirs A, B, and C to a single delivery point J (for Junction), where a total discharge Q is to be delivered. Shown in the figure are also the elevations of the free surface of the reservoirs, namely, H_A, H_B, and H_C, as well as the piezometric head elevation of the junction point, H_J.

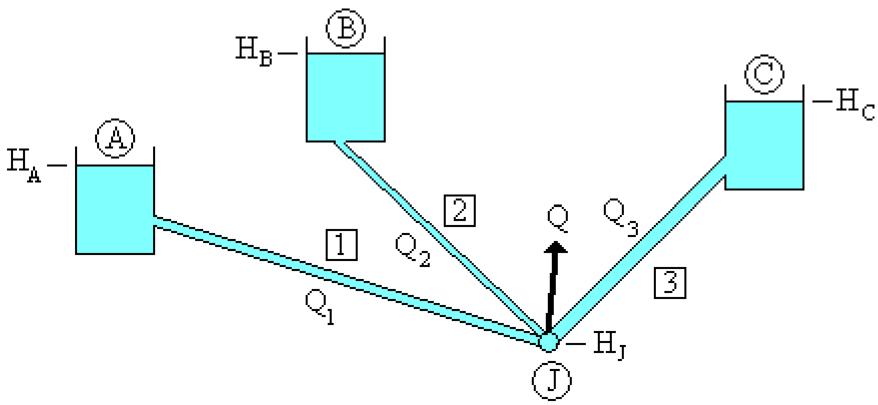


Figure 60. Three pipelines converging to a single delivery point.

Neglecting local or minor losses, the energy equations for the three pipelines shown above are written as:

$$H_A - H_J = (h_f)_1$$
$$H_B - H_J = (h_f)_2 \qquad \textbf{[Eq. 191]}$$
$$H_C - H_J = (h_f)_3$$

Where $(h_f)_1$, $(h_f)_2$, and $(h_f)_3$ are the friction losses in pipelines [1], [2], and [3], respectively.

A typical problem for the system illustrated in Figure 60 consists in determining the discharge Q given the elevations of the free surfaces in reservoirs A, B, and C (H_A, H_B, H_C) and that of point J (H_J). The individual pipe discharges are summed to obtain the total. The examples using Manning's equation and the Hazen-Williams formula presented below neglect local losses.

Example 75- Converging pipelines using Manning's equation

The system of Figure 60 has pipelines lengths, diameters, and Manning's n coefficients: $L_1 = 200\ ft$, $L_2 = 400\ ft$, $L_3 = 150\ ft$, $D_1 = 1.00\ ft$, $D_2 = 1.50\ ft$, $D_3 = 1.00\ ft$, $n_1 = 0.012$, $n_2 = 0.018$, and $n_3 = 0.010$. The elevations of interest are $H_A = 280\ ft$, $H_B = 290\ ft$, $H_C = 310\ ft$, and $H_J = 250\ ft$. Determine the total discharge delivered to junction J.

The energy losses in each pipeline are calculated as follows:

$$(h_f)_1 = H_A - H_J = 280\ ft - 250\ ft = 30\ ft,$$
$$(h_f)_2 = H_B - H_J = 290\ ft - 250\ ft = 40\ ft,$$
$$(h_f)_3 = H_C - H_J = 310\ ft - 250\ ft = 60\ ft.$$

As in example 73, Manning's equation is used to calculate the individual discharges as follows:

$$Q_1 = 0.4632 \cdot \frac{D_1^{\ 8/3}}{n_1} \cdot \sqrt{\frac{(h_f)_1}{L_1}} = 0.4632 \times \frac{1.00^{8/3}}{0.012} \times \sqrt{\frac{30}{200}} = 14.95\,cfs$$

$$Q_2 = 0.4632 \cdot \frac{D_2^{\ 8/3}}{n_2} \cdot \sqrt{\frac{(h_f)_2}{L_2}} = 0.4632 \times \frac{1.50^{8/3}}{0.018} \times \sqrt{\frac{40}{400}} = 23.99\,cfs$$

$$Q_3 = 0.4632 \cdot \frac{D_3^{\ 8/3}}{n_3} \cdot \sqrt{\frac{(h_f)_3}{L_3}} = 0.4632 \times \frac{1.00^{8/3}}{0.010} \times \sqrt{\frac{60}{150}} = 29.29\,cfs$$

The total discharge is *Q = 14.95 cfs + 23.99 cfs + 29.29 cfs = 68.23 cfs*

As in the previous parallel pipes example 73, using Manning's formula for friction losses, discharge calculations may also be made for individual pipes with *Pipe Flow, NRCS Hydraulics Formula* program. The discharges calculated will be slightly less than those shown above because *Pipe Flow, NRCS Hydraulics Formula* automatically accounts for a local loss at the pipe exit ($K_d = 1.0$).

Example 76 - Converging pipelines using Hazen-Williams formula

The system of Figure 60 has pipelines lengths, diameters, and Hazen-Williams coefficients: $L_1 = 200\ ft$, $L_2 = 400\ ft$, $L_3 = 150\ ft$, $D_1 = 1.00\ ft$, $D_2 = 1.50\ ft$, $D_3 = 1.00\ ft$, $C_{HW1} = 100$, $C_{HW\,2} = 80$, and $C_{HW\,3} = 120$. The elevations of interest are $H_A = 280\ ft$, $H_B = 290\ ft$, $H_C = 310\ ft$, and $H_J = 250\ ft$. Determine the total discharge delivered to junction *J*.

The energy losses in each pipeline are calculated as follows:

$$(h_f)_1 = H_A - H_J = 280\ ft - 250\ ft = 30\ ft,$$
$$(h_f)_2 = H_B - H_J = 290\ ft - 250\ ft = 40\ ft,$$
$$(h_f)_3 = H_C - H_J = 310\ ft - 250\ ft = 60\ ft.$$

As in example 74, the Hazen-Williams formula is used to calculate the individual pipe discharges as follows:

$$Q_1 = 0.432 \cdot C_{HW\,1} \cdot D_1^{\ 2.63} \cdot \left(\frac{(h_f)_1}{L_1}\right)^{0.54} = 0.432 \times 100 \times 1.00^{2.63} \times \left(\frac{30}{200}\right)^{0.54} = 15.50\,cfs$$

$$Q_2 = 0.432 \cdot C_{HW\,2} \cdot D_2^{\ 2.63} \cdot \left(\frac{(h_f)_2}{L_2}\right)^{0.54} = 0.432 \times 80 \times 1.50^{2.63} \times \left(\frac{40}{400}\right)^{0.54} = 28.95\,cfs$$

$$Q_3 = 0.432 \cdot C_{HW\,3} \cdot D_3^{\ 2.63} \cdot \left(\frac{(h_f)_3}{L_3}\right)^{0.54} = 0.432 \times 120 \times 1.00^{2.63} \times \left(\frac{60}{150}\right)^{0.54} = 31.60\,cfs$$

The total discharge is *Q = 15.50 cfs + 28.95 cfs + 31.60 cfs = 76.05 cfs*.

AGPipe design software for irrigation and livestock pipe systems, is available at:

http://www.wsi.nrcs.usda.gov/products/W2Q/water_mgt/Irrigation/irrig-mgt-models.html

AGPipe software performs hydraulic calculations and contains databases of local loss factors and friction factors, which may be populated by the user.

NRCS is also developing the Pipeline Design Tool for future field office use.

0332.4 Pipeline Networks

Pipe networks are utilized to supply water for urban and rural domestic and industrial uses. Networks are also used by irrigation districts and occasionally in livestock watering systems. A pipe network is simply a collection of pipes connected in a given geometric pattern and having at least one supply point and one delivery or consumption point. The points where individual pipelines join each other are known as *junctions* or *nodes*. Pipe networks are schematized as geometric constructs of lines representing the component pipes.

The main purpose of analyzing a given pipe network is to determine the discharge in each pipe and the piezometric head at each node. These results can be used then to verify that certain design guidelines, such as minimum flow velocities and minimum junction pressures, are satisfied. The analysis of pipe networks requires the simultaneous solution of a number of mathematical equations that represent equations of continuity at the nodes, and equations of energy around closed loops and/or pseudo-loops.

Analyses of simple pipe networks may be performed with a spreadsheet application, but spreadsheet use would become very tedious as the number of pipelines and loops increase. There are a number of publicly-available and commercial software that can be used for network solution in a more efficient manner, such as the U.S. Environmental Protection Agency's *EPANET* software.

0333 Appurtenances in Pipelines and Networks

In this section a number of appurtenances commonly used in pipelines and pipe networks are described.

0333.1 Air Vacuum and Release Valves

Entry and entrapment of air in a pipeline during filling and operation of a pipeline can cause development of air pockets. Air pockets tend to restrict and reduce flow or increase pumping costs in pipelines. Air pocket build-up in pumped pipelines can reduce flows by 5 to 15%. In low-head, gravity-driven flow, air pocket build-up can reduce flow up to 50%. On the other hand, a lack of air entry can cause pipe collapse if a vacuum develops behind flowing water as a pipeline empties. Air valves are of four types:

1. Air/vacuum relief valves, also known as kinetic air valves, large orifice air valves, vacuum breakers, low-pressure air valves, and air relief (not release) valves.

Large volumes of air are discharged before a pipeline is pressurized, especially at pipe filling. Large quantities of air are admitted when the pipe drains and at the appearance of water column separation.

2. Continuous acting air and vacuum valves, also known as double orifice air valves, or combination air valves, fill the functions of the air/vacuum relief valves and air release valves, admitting and releasing large quantities of air when needed, and releasing air continuously when the lines are pressurized.

3. Air release valves are also known as automatic air valves, small orifice air valves, continuous acting air vents, and pressure air valves. These vents continue to discharge air, usually in smaller quantities, after the air vacuum valves close, as the line is pressurized.

4. Vacuum relief valves are large orifice valves for vacuum relief only. These valves allow air to enter the pipeline.

Guidelines for placement and sizing of air valves:

1. Locate air release valves or continuous-acting air and vacuum valves at all high points and at 1/4 mile intervals on pipelines with constant grades. Locate an air release valve at the end of the line.

2. Aviod oversizing air release valves to lessen the possibility of water hammer. See section 0334 for more information on water hammer.

3. Locate an air-and-vacuum valve or vacuum valve to allow air to enter behind the water as a pipeline is emptied. Vacuum valves are generally not needed on pipelines of less than 3-inch diameter for protection from collapse; however, they may be needed to insure complete drainage of the line. On the other hand, vacuum valves are important on low head plastic irrigation pipe and large diameter steel pipe (>24”) to prevent collapse of the line.

Valve sizes may be determined for a pipeline by calculating: (1) flow for both filling and emptying the pipeline, (2) pipe collapse pressure, and (3) valve intake or discharge at a prescribed pressure across the valve as illustrated below.

Example 77 – Air valve sizing

Given: 18” diameter, 80 psi, SDR 51, PVC pipe, on a slope of 0.05 ft/ft. Note that SDR is the pipe outside diameter divided by the wall thickness.

a. Determine flow for draining the line using the Hazen-Williams formula (equation 169). C_{HW} for PVC pipe is 150:

$$Q = 0.432 \cdot 150 \cdot 1.5^{2.63} \cdot (.05)^{0.54} = 37.34\ cfs$$

Valves must re-enter air at the same rate as the calculated water flow of 37.34 cfs.

b. Determine pipe collapse pressure using equation for PVC pipe given in ASTM:

$P_c = [2E/(1-\nu^2)] \times [[1/[d_o/t] [(d_o/t)-1]^2]]$

Where:

P_c = collapse pressure of PVC pipe (psi)

E = Young's modulus of elasticity (400,000 psi)

ν = Poisson's ratio (0.33)

t = wall thickness (in.) d_o = outside diameter of pipe (in.)

$P_c = [(2)(400{,}000)/(1-0.33^2)] [[1/(51) [(51)-1]^2]] = 7$ psi

Use the collapse pressure determined above or 5 psi, whichever is lower (standard design practice).

c. Determine orifice diameter, using orifice equation (equation 198):

$$Q = C_d A \sqrt{2gh}$$

where C_d = flow coefficient = 0.6

A = orifice area

h = P_c (from above)/ω_{air} where: specific weight of air = ω_{air} = .0764 lbs/ft^3

P_c = (5 lbs/in^2) (144 in^2/ ft^2) = 720 psf

Rearranging the orifice formula, $A = Q/[C_d (2gh)^{.5}]$

$A = 37.34/[0.6 [2(32.2)(720/.0764)]^{.5}]$
$= 0.080$ ft^2

$A = (.080\text{ ft}^2)(144\text{ in}^2/\text{ft}^2) = 11.52\text{ in}^2$

$D = (4A/\pi)^{.5} = [(4)(11.52)/\pi]^{.5}$
= 3.83 in, orifice diameter

d. Determine the size of orifice required when filling the pipe. When filling a line, good practice is not to exceed 1 fps (note: not all systems provide that much control of filling velocity, see next part (e) of this example). Using a filling velocity of 1 fps:

$Q = AV$
$Q = (\pi/4)\ (1.5^2)\ (1) = 1.77$ cfs

Therefore, air must be discharged from the line at a rate of 1.77 cfs

Size the orifice for 2 psi pressure across the valve (standard design practice).

Using the orifice formula, as before:

$P_c = (2 \text{ lbs/in}^2)\ (144 \text{ in}^2/\text{ft}^2) = 288$ psf

$A = 1.77/[0.6\ [2(32.2)(288/.0764)]^{.5}\ ] = 0.006 \text{ ft}^2$
$A = (0.006 \text{ ft}^2)\ (144 \text{ in}^2/\text{ft}^2) = 0.86 \text{ in}^2$

$D = (4A/\pi)^{.5} = [(4)(0.86)/\pi]^{.5} = 1.05$ in. orifice diameter.

e. Alternative determination of orifice size required when filling the pipe. When filling a line, with an electric powered pumping plant not equipped with special controllers or valves, it may not be possible to limit filling velocity to 1 fps. The electric motor is on or off which might produce filling velocities of up to 5 fps. Using a filling velocity of 5 fps:

$Q = AV$
$Q = (\pi/4)\ (1.5^2)\ (5) = 8.84$ cfs

Therefore, air must be discharged from the line at a rate of 8.84 cfs

Size the orifice for 2 psi pressure across the valve (standard design practice).

Using the orifice formula, as before:

$P_c = (2 \text{ lbs/in}^2)\ (144 \text{ in}^2/\text{ft}^2) = 288$ psf

$A = 8.84/[0.6\ [2(32.2)(288/.0764)]^{.5}\ ] = 0.03 \text{ ft}^2$
$A = (0.03 \text{ ft}^2)\ (144 \text{ in}^2/\text{ft}^2) = 4.32 \text{ in}^2$

$D = (4A/\pi)^{.5} = [(4)(4.32)/\pi]^{.5} = 2.35$ in. orifice diameter

Comparing orifice sizes of 3.83 in. diameter for vacuum relief and 1.05 or 2.35 in. diameter for air release shows that the 3.83 in. diameter orifice should be used if an air-and-vacuum valve, or continuous-acting air-and-vacuum valve, is to be used. If individual air release and vacuum relief valves are used, then they each may be of the size determined.

Additional information on the operation and settings for air valves is available from the valve manufacturers.

0333.2 Air Vents

Air vents act similarly to air vacuum or air release valves, however, they allow for the release of large volumes of air near pumps or water intakes, pressure boxes, and check valves. Air vents are often used in low-head pipelines. The figure below shows a simple air vent consisting of a cylindrical chamber of diameter D_c located at a distance L from a water intake where air entrainment occurs. The height of the air vent chamber should be at least one-half the diameter of the pipe ($D/2$), and the chamber's area should be at least half of the mainline pipe cross sectional area. For a cylindrical chamber, the minimum diameter is, therefore,

$$D_c = \sqrt{\frac{D^2}{2}} \qquad \textbf{[Eq. 192]}$$

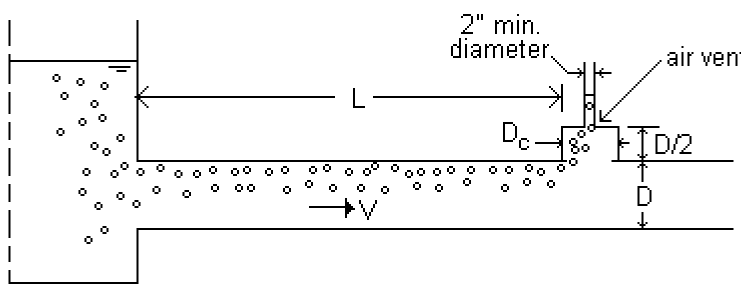


Figure 61. Location and dimensions of air vent chamber near a water intake.

The minimum length L, measured from the pipe entrance is given by, Roberson, et.al.(1956):

$$L = 1.76 \cdot V \cdot D \qquad \textbf{[Eq. 193]}$$

Where: V is the flow velocity in the pipeline. This equation uses V in *fps*, D in *ft*, and L in *ft*.

Using units of the International System (S.I.), the corresponding equation is:

$$L = 5.77 \cdot V \cdot D \qquad \textbf{[Eq. 194]}$$

where V is in *m/s*, D in *m*, and L in *m*.

Example 78 – Air vent chamber sizing

A 12-in diameter pipeline is to carry a discharge Q = 2.5 cfs. Determine the minimum diameter, height, and location of an air vent chamber from a water intake where air entrainment occurs.

The pipe diameter is D = 1 ft, and the flow velocity can be calculated from equation 31:

$$V = \frac{4Q}{\pi D^2} = \frac{4 \times 2.5\, ft^3 / s}{3.1416 \times (1\, ft)^2} = 3.18\ fps$$

The chamber should have a height of D/2 = 0.5 ft = 6 in, and a minimum diameter of:

$$D_c = \sqrt{\frac{D^2}{2}} = \sqrt{\frac{(1\ ft)^2}{2}} = 0.707\ \ ft$$

The distance from the water intake to the air vent should be at least:

$$L = 1.76 \cdot V \cdot D = 1.76 \times 3.18 \times 1 = 5.60\ ft$$

0333.3 Pressure Control Valves

Pressure relief valves (or pressure safety valves) are used in pipeline systems to protect against excessive pressure. They are designed to open and discharge small amounts of water at a preset pressure limit. Excess pressure would open the valve and allow some water to be released.

Pressure reducing valves are used where a predetermined lower pressure is necessary for the proper function of certain components such as emitters in microirrigation systems. They can be used in systems with variable pressure to maintain a lower constant pressure downstream of the valve.

Pressure-sustaining valves consist of the basic valves and a three-way pressure-sustaining pilot. Pressure is sustained at the upstream of the valve to a preset level, while the valve outlet drains excessive pressure in order to maintain the preset inlet pressure. Pressure is maintained constant, regardless of upstream fluctuating pressure and flow rate. Pressure sustaining valves are used on hilly terrain to maintain pressure in elevated areas and many other applications where sustained pressure is necessary

Details on the operation of pressure relief, reducing, and sustaining valves are available from the valve manufacturers. The pipeline system pressure range and capacity are parameters needed for valve selection.

Example 79 – Pressure relief valve selection

Given: 8” diameter, 80 psi, SDR 51, plastic irrigation pipe. Pipe inside diameter is 7.84 in = 0.65 ft. Design pipeline velocity is 5 fps.

Select a pressure relief valve to protect against possible surge pressure at the end of the pipeline. The valve is set to relieve pressure at 80 psi, which is the pipe pressure rating. Size the pressure relief valve to pass the full flow at 150% of the pipe pressure rating:

First, determining the pipeline flow:

$Q = AV$
$Q = (\pi/4)\ (0.65^2)\ (5) = 1.66$ cfs

Determining minimum orifice diameter, using orifice equation (equation 198):

$$Q = C_d A\sqrt{2gh}$$

where C_d = flow coefficient = 0.6

A = orifice area

$P = 1.5\ (80\ \text{lbs/in}^2)\ (144\ \text{in}^2/\text{ft}^2) = 17280$ psf

$h = P/\omega_w$ where: specific weight of water = $\omega_w = 62.4\ \text{lbs/ft}^3$

Rearranging the orifice formula, $A = Q/[C_d\ (2gh)^{.5}]$

$A = 1.66 / [0.6\ [2(32.2)\ (17280/62.4)]^{.5}]$
$= 0.0207\ \text{ft}^2$

$A = (.0207\ \text{ft}^2)\ (144\ \text{in}^2/\text{ft}^2) = 2.98\ \text{in}^2$

$D = (4A/\pi)^{.5} = [(4)(2.98)/\pi)]^{.5}$

= 1.95 in, orifice diameter

Review of manufacturer’s literature indicate that pressure relief valves with 2-in and larger orifices are available. If, for safety concerns, the designer wishes to reduce the valve escape velocity, larger-size valves are available or multiple valves may be used. Find the dimensions of the valve opening using the continuity equation: $A = Q/V$.

Example 80 – Pressure reducing valve selection

Given: 3" diameter, SDR 41, PVC pipe. Pipe inside diameter is 3.33 in = 0.278 ft. The pipe is a 500-ft sprinkler lateral, laid on a 2.5% uphill slope. The lateral's design velocity is 5 fps. A 30 psi pressure is needed at the last (end-of-line) sprinkler. Select a pressure reducing valve to maintain a maximum of 20% pressure variation at the sprinklers located along the lateral.

a. First, determine the pipeline flow:

$Q = AV$
$Q = (\pi/4)\,(0.278^2)\,(5) = 0.303$ cfs

b. Determine friction losses for the lateral using the Hazen-Williams formula (equation 171). C_{HW} for the PVC pipe lateral is 130.

$$h_f = 4.732\left(\frac{0.303}{130}\right)^{1.85}\frac{500}{0.278^{4.87}} = 16.27\ ft$$

The friction loss of 16.27 ft is equivalent to 7.0 psi.

c. Determine minimum lateral pressure needed upstream of the first sprinkler:

This pressure is the sum of the pressure needed at the last sprinkler plus the lateral elevation difference plus the friction losses (local losses neglected).

Elev. Diff. = (0.025 ft/ft) (500) = 12.5 ft, equivalent to 5.4 psi

$P = 30 + 5.4 + 7.0 = 42.4$ psi

At the first sprinkler, the pressure needs to be reduced from about 43 psi to at least 36 psi (1.2 x 30psi). Review of manufacturer's literature indicates that a 2-in pressure reducing valve is available to meet the need for reduced pressure. To maintain a maximum of 20% sprinkler pressure variation and 30 psi at lateral end, pressure reducing valves are needed at about the lower half of the lateral's sprinklers.

0333.4 Surge/Air Chambers

Surge or air chambers are vertical chambers, typically of cylindrical cross-section and open to the atmosphere, attached to pipelines to allow pressure relief during hydraulic transients. They perform a similar role as pressure relief valves, except that instead of releasing water flow to another pipeline, they allow the water surface level in the chamber to increase in response to a local pressure increase. During a hydraulic transient (unsteady flow) the water surface in the chamber will oscillate until a steady state is recovered. Figure 62, below, shows a schematic of a surge chamber in a pump-pipeline system. The surge chamber in this case would help dissipate a pressure wave that may result after a sudden failure of the pump. The system also includes a check valve that shuts off flow as the flow reverses.

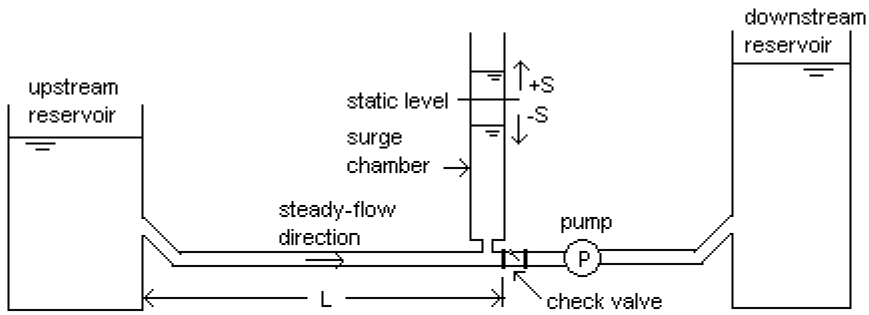


Figure 62. Schematic of pump-pipeline system protected with a surge chamber.

For the situation illustrated in the figure above, if the cross-sectional area of the surge chamber is A_c and that of the pipeline is A, the maximum height S of the water surface in the chamber above its steady-state level (i.e., the amplitude of the oscillation) is calculated as:

$$S = \frac{Q}{A_c}\left(\frac{\pi}{2}\right)\sqrt{\frac{A_c \cdot L}{A \cdot g}} \qquad \textbf{[Eq. 195]}$$

where Q is the steady-state discharge, L is the length of pipeline between the upstream reservoir and the check valve, and g is the acceleration of gravity $(= 9.81\ m/s^2 = 32.2\ ft/s^2)$. The time in seconds required for the water level in the chamber to first reach its maximum height (one quarter of the period of oscillation) is calculated as, Tullis (1989):

$$T = \frac{\pi}{2}\sqrt{\frac{A_c \cdot L}{A \cdot g}} \qquad \textbf{[Eq. 196]}$$

The water in the chamber will oscillate until friction in the pipe, and in the chamber, slows down the oscillation and produces a complete flow shut off in the pipe upstream of the check valve. Notice that in the absence of a check valve, flow reversal will occur through the pump which could affect its operation after start up.

Example 81 – Surge chamber calculation

The pipeline in Figure 62 is a *1-ft* diameter pipeline and the length from the supply reservoir to the check valve is *300 ft*. If the pipe carries a steady-state discharge of *4 cfs*, determine the maximum water surface increase in a surge chamber of cross-sectional area $A_c = 0.785\ ft^2$. If the static level at the chamber is $H_0 = 500\ ft$, how high will the water level reach in the surge tank? What is the time T required to reach the maximum water elevation?

With D = 1.0 ft, the pipe area is:

$$A = \pi D^2/4 = 3.146 \times (1\ ft)^2 / 4 = 0.785\ ft^2$$

With L = 300 ft, Q = 4 cfs, g = 32.2 ft/s^2, then:

$$S = \frac{Q}{A_c}\left(\frac{\pi}{2}\right)\sqrt{\frac{A_c \cdot L}{A \cdot g}} = \frac{4\,ft^3/s}{0.785\,ft^2}\left(\frac{\pi}{2}\right)\sqrt{\frac{0.785\,ft^2 \times 300\,ft}{0.785\,ft^2 \times 32.2\,ft/s^2}} = 24.43\,ft$$

Thus, the maximum water level is:

$$H = H_0 + S = 500\ ft + 24.43\ ft = 524.43\ ft$$

The time required to reach the maximum water level:

$$T = \frac{\pi}{2}\sqrt{\frac{A_c \cdot L}{A \cdot g}} = \frac{3.1416}{2}\sqrt{\frac{0.785\,ft^2 \times 300\,ft}{0.785\,ft^2 \times 32.2\,ft/s^2}} = 4.79\,s$$

0333.5 Check Valves

Check valves are valves that allow flow only in one direction. The pressure of the flowing water opens the valves in the preferred direction. A sudden change in flow direction, such as the case of a pump failure, immediately closes the valve avoiding a reverse flow. The lack of a check valve leaves a pump unprotected against backflow, which may damage the pump and affect its operational efficiency. Details on the operation of check valves can be obtained from the valve manufacturers.

0334 Hydraulic Transients (Water Hammer)

The term hydraulic transient refers to a pipeline flow, and its attendant pressure, changing rapidly with time. A hydraulic transient in a pipeline can be originated by the closing of a valve, a pump failure, or a sudden increase in pipeline demand. Water hammers can easily be produced by a sudden closure of a valve. In this case, a pressure wave originates at the valve that travels through the pipe at speeds of the order of 1,440 m/s or 4,700 ft/s. The pressure wave reflects at the other end of the pipeline, be it a reservoir or a main line connection, and travels back towards the valve where pressure builds up again. The process repeats many times until friction dampens the pressure wave and the water flow finally stops. Typically a hammering noise is produced by the transient, thus the name *water hammer*. Pipelines, appurtenances, and tanks subject to water hammer may suffer deformation and damage in joints and other locations. In designing pipelines, care should be taken to prevent the presence of the water hammer phenomenon by providing check valves, pressure relief valves, and surge chambers in the pipeline systems.

Valve closure hydraulic transients can be minimized by closing the valve slowly. Unexpected hydraulic transients, such as a pump failure, will likely produce a high-magnitude pressure wave in pipelines with its attendant vibration. To prevent damage to the pipeline in such a case the pipeline should be provided with valves or surge chambers. The pipeline will be able to better survive a sudden hydraulic transient if it is buried, or otherwise anchored securely.

The formal analysis of hydraulic transients in a pipeline includes the simultaneous solution of two partial differential equations involving the discharge Q and the total head H at different locations x along the pipeline and at different times t. This type of analysis requires complex computer programming, considered beyond the scope of this handbook.

The closing of a valve is a common cause of hydraulic transients. If closing a valve changes the flow velocity in a pipe from V to V_f, the instantaneous increase in pressure near the valve, Δp, is calculated by using the equation:

$$\Delta p = (V - V_f)\sqrt{\frac{\omega / g}{\frac{1}{E_v} + \frac{D}{t \cdot E}}} \qquad \textbf{[Eq. 197]}$$

Where ω is the specific weight of water (typically, $\omega = 62.4\ lb/ft^3$), g is the acceleration of gravity (= $32.2\ ft/s^2$), E_v is the bulk modulus of elasticity of water, t is the pipe wall thickness, and E is the modulus of elasticity for the pipe material. A typical value of the modulus of elasticity for water is $E_v = 311{,}000\ psi$ at 60°F (see Exhibit 4). Modulus of elasticity values for common pipeline materials include steel ($E = 30{,}000{,}000\ psi$), cast iron ($E = 15{,}000{,}000\ psi$), concrete ($E = 3{,}000{,}000\ psi$) and PVC ($E = 400{,}000\ psi$, *with some variation by manufacturing class*).

For additional information on water hammer analyses see Finnemore and Franzini (2002) or USDA (2005).

Example 82 – Pressure increase with sudden closing of a valve

Consider a steel pipe (E = 30,000,000 psi) with a wall thickness $t = 0.25\ in = 0.25/12\ ft = 0.02\ ft$, and a diameter $D = 0.5\ ft$. Closing a valve in a pipe that reduces the flow velocity from V = 2.5 fps to V_f = 0.5 fps for water at 60°F in such a pipeline produces a pressure increase of:

$$\Delta p = (2.5\,fps - 0.5\,fps)\sqrt{\frac{(62.4\,lb/ft^3)/(32.2\,ft/s^2)}{\cdot\frac{1}{311000 \times 144 lb/ft^2} + \frac{0.5\,ft}{0.02\,ft \times 30000000 \times 144 lb/ft^2}}}$$

$$\Delta p = 16{,}604\ lb/ft^2 = 16{,}604/144\ psi = 115\ psi$$

The typical value for atmospheric pressure is *14.7 psi*, thus, an increase of *115 psi* represents almost eight times the atmospheric pressure. Pipe should have adequate pressure ratings or otherwise be protected to withstand hydraulic transients.

0335 Cavitation

When the local absolute pressure in a closed conduit (by virtue of its elevation and the hydraulic characteristics of the system) falls below the vapor pressure of water, vapor bubbles, or cavities, can form in the flow. These cavities are then carried with the flow towards zones of higher pressure where they implode (collapse onto themselves) producing a characteristic noise, vibration, and even pitting of the pipeline walls at the point of bubble collapse. Cavitation conditions are to be avoided because of their damaging effects, since cavitation can cause pitting and perforations in pipes, and even collapsing of pipes if the pressure drop at a given location is excessively large. Locations where cavitation may occur are the high point of a pipeline, or the suction side of a pump in a pipeline system. Cavitation is undesirable for pump operation since it can reduce pump efficiency and cause deterioration of the impeller. An example of cavitation analysis in a siphon is presented below. Cavitation was introduced in section 0304 Physical Properties of Water, in relation to the vapor pressure of water.

Example 83 – Cavitation at high point of a siphon

Figure 63 below illustrates a conduit delivering water from reservoir (A) to reservoir (B). Such an arrangement, where the middle part of the conduit raises above the free surface of both reservoirs is called a siphon. To start the siphon, it is necessary to fill the conduit with water by using, for example, a vacuum pump at point (C). Once the water flow is established the vacuum pump can be removed and the system continues flowing as long as the free surface at reservoir (A) is higher than that at reservoir (B). Siphon systems can be set up to overcome a hill separating two reservoirs or to discharge water downstream over a dam.

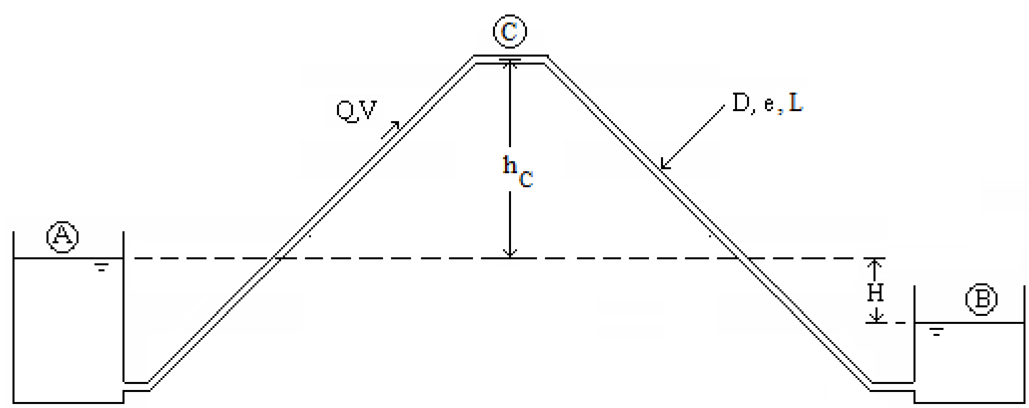


Figure 63. Siphon conduit connecting two reservoirs.

Suppose that the siphon has a diameter *D = 1.0 ft*, a length *L =200 ft*, and the material is smooth enough that the roughness can be taken as *e = 0*. Let the difference of elevation between the reservoirs be *H = 20 ft*, and the elevation difference of point (C) above the level of reservoir (A) be $h_C = 10$ *ft*. Local loss coefficients are $K_e = 0.5$ and $K_d = 1.0$ for the entrance and discharge points at reservoirs (A) and (B), respectively. (a) Determine the discharge in the siphon. (b) Determine the absolute pressure at high point (C), located half way through the conduit, and check if cavitation is likely to occur. Assume that the water temperature is 60^oF, and use an atmospheric pressure p_{atm} = 14.7 psi. (c) What is the maximum value of h_C to prevent cavitation from taking place at point (C)?

(a) Writing the energy equation between points (A) and (B), with $V_A = V_B = 0$, using gage pressures $p_A = p_B = 0$, taking $z_A = H$ and $z_B = 0$, and including friction (using the Darcy-Weisbach equation to estimate friction losses) and local losses ($\Sigma K = 1.5$), the resulting equation is:

$$H = \frac{8Q^2}{\pi^2 g D^4}\left(f\left(\frac{e}{D}, \frac{4Q}{\pi \nu D}\right)\cdot \frac{L}{D} + \Sigma K \right)$$

Exhibit 4 gives the kinematic viscosity, $\nu = 1.217x10^{-5}\ ft^2/s$, for water at 60°F. A spreadsheet application gives $Q = 14.73\ cfs$.

(b) To determine the absolute pressure p_B at point (C) write the energy equation between points (A) and (C):

$$z_A + \frac{p_A}{\omega} + \frac{V_A^2}{2g} - h_f - \Sigma h_L = z_C + \frac{p_C}{\omega} + \frac{V_C^2}{2g}$$

With $V_A = 0$, gage pressure $p_A = 0$, and elevations:

$$z_A = H = 20\ ft$$

$$z_C = H + h_C = 20\ ft + 10\ ft = 30\ ft$$

The flow velocity at point (C) is the constant value:

$$V = V_C = \frac{4Q}{\pi D^2} = \frac{4 \times 14.73\, ft^3/s}{3.1416 \times (1.0\, ft)^2} = 18.75\, ft/s$$

And the Reynolds number is:

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{18.75\, ft/s \times 1.0\, ft}{1.217 \times 10^{-5}\ ft^2/s} = 1.54 \times 10^6$$

With this value of the Reynolds number (indicating turbulent flow) and relative roughness e/D = 0, the Swamee-Jain equation (equation 162) produces the following friction factor:

$$f = \frac{0.25}{\log^2\left(0.27\cdot\frac{e}{D}+4.62\cdot\left(\frac{\nu D}{Q}\right)^{0.9}\right)} = \frac{0.25}{\log^2\left(0+4.62\cdot\left(\frac{1.217\times10^{-5}\,ft^2/s\times1.0\,ft}{14.73\,ft^3/s}\right)^{0.9}\right)}$$

$$f = 0.0108$$

The friction losses (using the Darcy-Weisbach equation) and local head losses (entrance losses) are calculated as follows:

$$h_f = f\cdot\frac{(L/2)}{D}\cdot\frac{V^2}{2g} = 0.0108\times\frac{(200\,ft/2)}{1.0\,ft}\times\frac{(18.75\,ft/s)^2}{2\times32.2\,ft/s^2} = 5.9\,ft$$

$$\Sigma h_L = \Sigma K\cdot\frac{V^2}{2g} = 0.5\times\frac{(18.75\,ft/s)^2}{2\times32.2\,ft/s^2} = 2.73\,ft$$

Substituting values calculated above into the energy equation:

$$20\,ft+\frac{0}{\omega}+\frac{0}{2g}-5.9\,ft-2.73\,ft = 30\,ft+\frac{p_C}{\omega}+\frac{(18.75\,ft/s)^2}{2\times32.2\,ft/s^2}$$

Solving for the pressure head at point (C):

$$\frac{p_C}{\omega} = -24.09\,ft$$

And, using $\omega = 62.4\ lb/ft^3$, the gage pressure at point (C) is:

$$p_C = 62.4\frac{lb}{ft^3}\times(-24.09\,ft) = -1503\,psf = -\frac{1503}{144}\,psi = -10.44\,psi$$

Calculating the absolute pressure at point (C) and comparing to the vapor pressure, $p_v = 0.26\ psi$, for water at 60^oF (see Exhibit 4):

$$(p_C)_{abs} = p_C + p_{atm} = -10.44\,psi + 14.7\,psi = 4.26\,psi > 0.26\,psi$$

Since the absolute pressure at point (C) is larger than the vapor pressure of water at a temperature of 60^oF, no cavitation would occur at point (C) for the siphon system shown.

(c) To determine the maximum height h_C to avoid cavitation at point (C), the absolute pressure at point (C) is set equal to the vapor pressure of water at 60^oF, and the value of h_C from the energy equation between points (A) and (C) is calculated.

The gage pressure at point (C) is:

$$p_C = \left(p_C\right)_{abs} - p_{atm} = p_V - p_{atm} = 0.26\,psi - 14.7\,psi = -14.44\,psi\,.$$

The corresponding pressure head is:

$$\frac{p_C}{\omega} = \frac{-14.44\,psi}{62.4lb/\,ft^3} = \frac{-14.44\,psi \times 144\,psf\ /\ psi}{62.4lb/\,ft^3} = -33.32\,ft\ .$$

The velocity head, head losses, and elevation of point A are the same as in the energy equation used above. The elevation of point (C) is:

$$z_C = (h_C)_{\max} + 20\,ft$$

Thus, the energy equation between (A) and (C) becomes:

$$20\,ft + \frac{0}{\omega} + \frac{0}{2g} - 5.9\,ft - 2.73\,ft = (h_C)_{\max} + 20\,ft - 33.32\,ft + \frac{(18.75\,ft/s)^2}{2 \times 32.2\,ft/s^2}$$

The resulting value of h_C is:

$$(h_C)_{max} = 19.23\ ft$$

0336 Culverts

A culvert is a relatively short conduit connecting two open channels and typically located under a road or highway. Figure 64 below illustrates three possible flow regimes when a culvert's inlet is submerged.

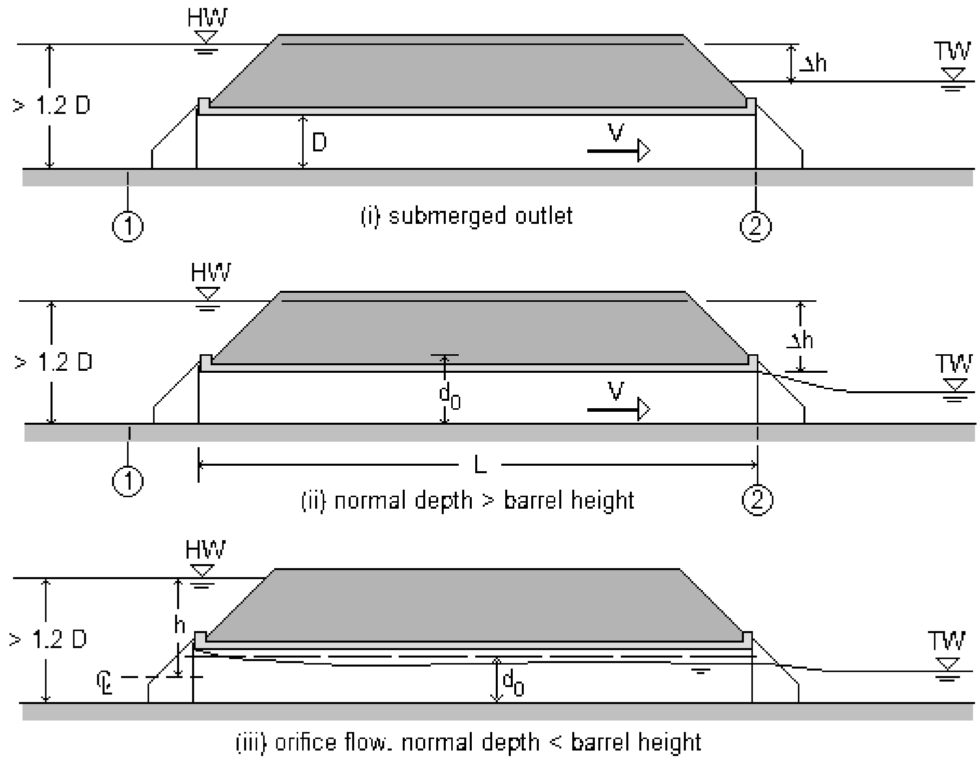


Figure 64. Flow regimes for submerged inlet flow in culverts.

The flows of Figure 64 (i) and 64 (ii) are said to be under *outlet control*, while that of 64 (iii) is under *inlet control*. In Figure 64, *V* stands for the flow velocity in the full-flowing culvert; HW stands for *head water* and *TW* for *tailwater* depths, i.e., the depths of flow upstream and downstream of the culvert. *D* is the diameter of the culvert, *h* is the vertical distance from the head water surface to the centerline of the culvert, and Δh is the difference between the headwater and tailwater depths.

Publication *FHWA-NHI-01-020*, *HYDRAULIC DESIGN OF HIGHWAY CULVERTS*, *Hydraulic Design Series No. 5*, 2001, (*FHWA, HDS 5*) available from the Federal Highway Administration (FHWA) provides detailed information for the analysis of culvert flow. This publication is available at:

http://www.cflhd.gov/design/hyd/hds5_03r.pdf

The FHWA also provides a computer program, *HY-8,* for calculating culvert flow. The program is available at:

http://www.fhwa.dot.gov/engineering/hydraulics/software/hy8/

0336.1 Culvert Flow with Inlet Control

This case is represented in Figure 64(iii). The capacity of the culvert is limited by the capacity of the culvert opening, rather than by conditions farther downstream. In such case, the open-channel normal depth of flow d_0 in the culvert is less than the barrel height

(height of the culvert section), and the entrance does not allow enough water into the culvert to allow filling the full barrel height. Thus, the flow is under inlet control. The discharge Q into the culvert can be calculated using the following equation (orifice flow):

$$Q = C_d A\sqrt{2gh} \qquad \textbf{[Eq. 198]}$$

Where A is the area of the inlet section, C_d is a discharge coefficient, g is the acceleration of gravity, and h is the smaller vertical distance from the HW surface to the centerline of the inlet or the HW and TW depth difference.

For inlet control, the required HW head is calculated as:

$$h = \frac{Q^2}{2gC_d^2 A^2} \qquad \textbf{[Eq. 199]}$$

Values of the discharge coefficient C_d vary from 0.62 (for a sharp-edged inlet without contraction suppression) to 1.0 (for a well-rounded inlet). Use the value of $C_d = 0.60$ as a conservative reference value. Contraction suppression refers to the absence of contracted flow at the inlet cross-section. Suppressed contraction at the bottom of the inlet occurs when the inlet invert is set at stream bed level. Partial contraction suppression can be achieved by using flared wingwalls at the inlet approach.

As an alternative to equation 198 (general orifice flow equation), *FHWA*, *HDS 5* includes a submerged inlet control equation, based on extensive laboratory testing.

Example 84 – Circular culvert calculation under inlet control

A circular culvert of diameter $D = 2\ ft$, carrying a discharge $Q = 40\ ft^3/s$, is to be designed under inlet control. What is the required headwater height, h, above the culvert centerline if the inlet discharge coefficient is $C_d = 0.75$.

The pipe area is:

$$A = \pi D^2/4 = 3.1416\times (2\ ft)^2 / 4 = 3.14\ ft^2$$

And the HW head is:

$$h = \frac{Q^2}{2gC_d^2A^2} = \frac{(40\,ft^3/s)^2}{2\times 32.2\,ft/s^2 \times 0.75^2 \times (3.1416\,ft)^2} = 4.48\,ft$$

Checking that HW depth exceeds 1.2D:

$$HW\ depth = 4.48\,ft + 1.0\,ft = 5.48\,ft > 1.2\times 2\,ft = 2.4\,ft$$

0336.2 Culvert Flow with Outlet Control

Figures 64(i) and 64(ii) show instances of flow under outlet control. In 64(i) the outlet is fully submerged, a condition that may result from inadequate channel capacity downstream or by an existing backwater from a connecting stream. Even if the tailwater depth is below the barrel height, as shown in Figure 64(ii), the normal flow depth d_0 within the culver is greater than the culvert height D, and the culvert flows full.

Analysis of flow for the outlet control cases of Figures 64(i) and 64(ii) is approached by writing the energy equation between the free surface of the headwater section (1) and that of the tailwater section (2). The elevation of the outlet (section 2) is taken as a reference, so that $z_1 = \Delta h$, and $z_2 = 0$. Using gage pressure, $p_1 = p_2 = 0$. Also, $V_1 = 0$, and $V_2 = V$. Including head losses in the culvert, the resulting energy equation is:

$$\Delta h = h_{:L} + \frac{V^2}{2g} \qquad \textbf{[Eq. 200]}$$

In the simplest case, the head losses h_L include friction head losses h_f and entrance losses h_e. Friction head losses are calculated using Manning's equation:

$$h_f = \frac{n^2 V^2 L}{C_u^2 R_h^{4/3}} \qquad \textbf{[Eq. 201]}$$

Where n is Manning's resistance coefficient, C_u is the coefficient in Manning's equation ($C_u = 1.0$ for units of the SI, and $C_u = 1.486$ for English units), and R_h is the hydraulic radius in the culvert cross-section. Entrance losses are calculated using:

$$h_e = K_e \cdot \frac{V^2}{2g} \qquad \textbf{[Eq. 202]}$$

A higher value for K_e, the entrance loss coefficient, gives a higher head loss. Values of the entrance loss coefficient are presented in Table 8, Section 0331.6 Local Losses in Pipelines. Also see *FHWA, HDS 5* for additional values of entrance loss coefficients.

Substituting the sum of equations 201 and 202 for h_L in equation 200 produces the equation:

$$\Delta h = \left(K_e + \frac{2gn^2 L}{C_u^2 R_h^{4/3}} + 1 \right) \cdot \frac{V^2}{2g} \qquad \textbf{[Eq. 203]}$$

The approach presented in *FHWA, HDS 5* includes the use of a bend coefficient K_b, and an exit coefficient K_x (to replace the value of 1 in equation 203); the resulting equation is:

$$Q = A\sqrt{\frac{2g\Delta h}{K_e + K_p L + K_b + K_x}} \quad \textbf{[Eq. 204]}$$

Where K_p is the pipe friction loss coefficient, which, in equation 203, is equal to:

$$K_p = \frac{2gn^2}{C_u^2 R_h^{4/3}} \quad \textbf{[Eq. 205]}$$

A typical culvert design consists in determining the required diameter of a culvert given the design discharge, the inlet entrance conditions, the pipe material and length, the culvert slope, and the headwater and tailwater depths. The use of *Culvert Flow, USDA-NRCS Hydraulics Formula* program is illustrated in the following example.

Example 85 – Circular culvert sizing

It is desired to install $L = 50$ *ft* of concrete culvert pipe, $n = 0.012$, in a drainage channel for a road crossing. The design discharge is $Q = 80$ *cfs* with a tailwater depth of $d_2 = 3.0$ *ft*. The slope of the culvert is to be *0.002 ft/ft*. The maximum headwater depth is $d_1 = 5.0$ *ft*. Determine the required pipe diameter, if the inlet has a groove edge and a headwall.

The elevation change in *50 ft* of culvert is:

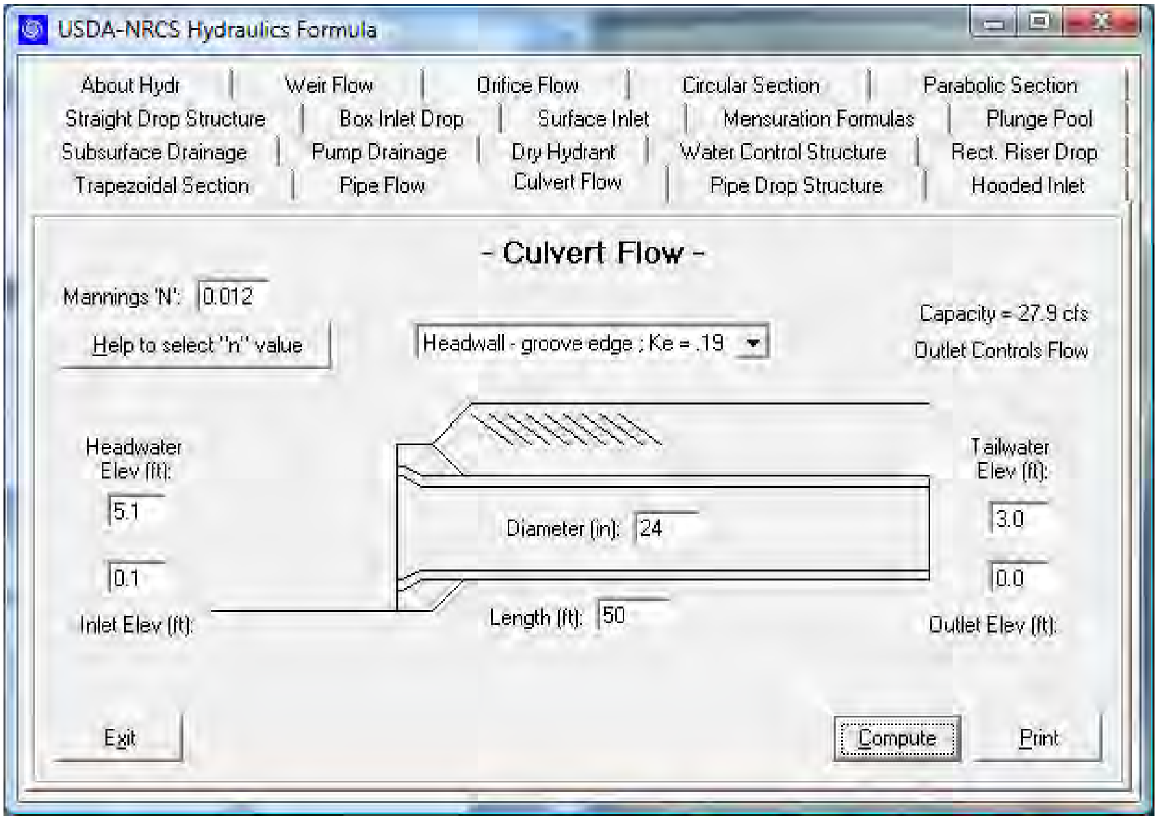
$$\Delta z = S_0 \cdot L = 0.002\ ft/ft \times 50\ ft = 0.1\ ft$$

Since the culvert outlet elevation is taken to be "0", the inlet elevation is *0.1 ft*.

The figure below shows the calculation of the discharge for culvert flow using the *Culvert Flow* tab in the *USDA-NRCS Hydraulics Formula* program for a guessed diameter of *24 in = 2 ft*. The result shows $Q = 27.9$ *cfs*, which is smaller than the required discharge of *80 cfs*. The diameter is increased until $Q = 80$ *cfs*. The following table shows the convergence of values:

D (in)	*Q (cfs)*
24	*27.9*
36	*67.1*
48	*105.7*
42	*88.0*
39	*78.7*
40	*81.8*

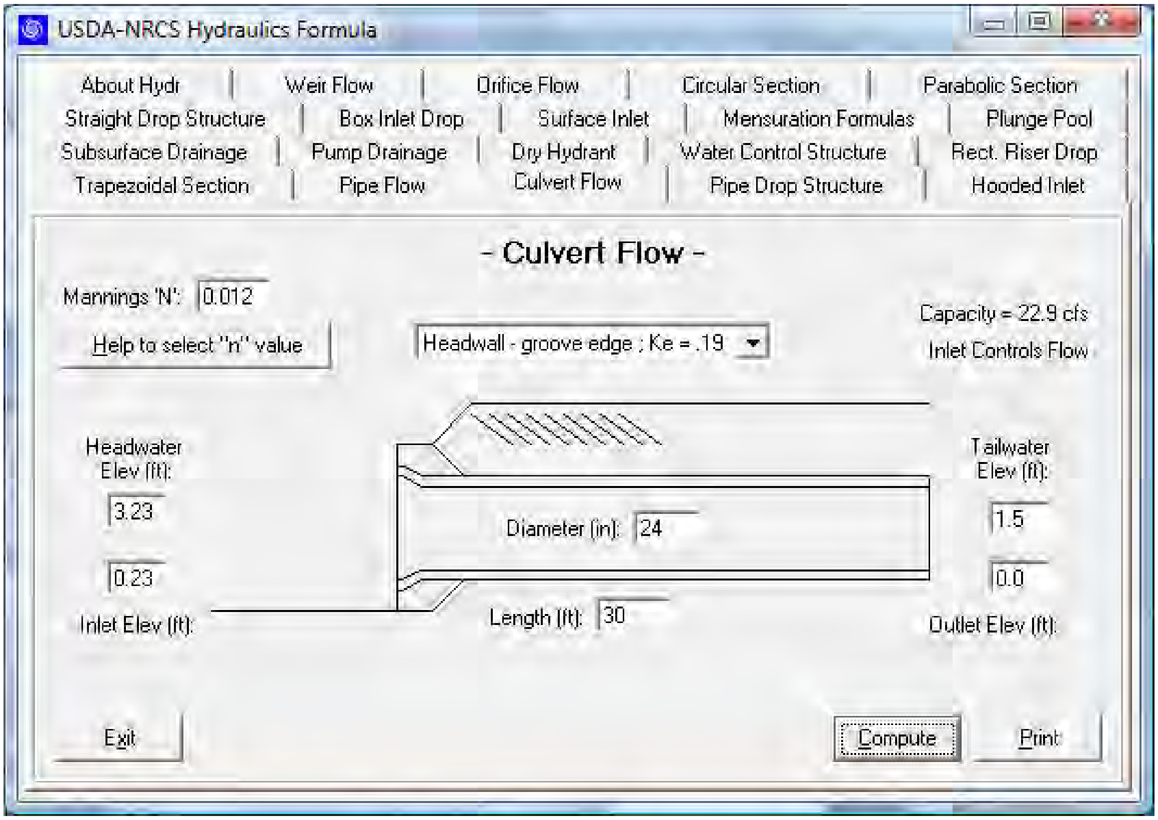
A standard sized concrete culvert of $D = 42$ *in* will carry the required *80 cfs*.



Example 86 - Culvert discharge calculation

Calculate the discharge in a PVC culvert with a diameter of 2 ft (24 in), and a length of 30 ft if the head wall inlet has a groove edge. The inlet invert is located 0.23 ft above the outlet invert (set as zero), the headwater depth is 3.0 ft (headwater elevation = 3.23 ft), while the tailwater depth is 1.5 ft.

The *USDA-NRCS Hydraulics Formula* program aids in the selection of the values of Manning's *n* and the entrance coefficient K_e. The discharge calculation is shown below:



Thus, the calculated discharge is $Q = 22.9$ *cfs* and the inlet controls the flow (the equations associated with inlet control apply, see section 0336.1).

In some applications the culvert headwater depth is below the barrel height producing *free or unsubmerged inlet conditions*. The techniques of open channel flow presented in section *0324.1 Gradually Varied Flow* can be used to solve for the critical depth of flow and the headwater depth. This analysis can allow for entrance and exit losses. As an alternative, *FHWA, HDS 5* includes an unsubmerged inlet control equation, based on extensive laboratory testing.

An alternative for culvert flow calculations is the use of nomograms. Exhibit 7 shows examples of culvert flow calculations using nomograms for both inlet and outlet control.

0337 Sprinkler Irrigation

Sprinkler systems irrigate by spraying water in a desirable pattern. Sprinkler systems typically include a pump, a main pipeline (or, simply, main), lateral pipelines (or, simply, laterals), risers, and sprinkler heads.

Sprinkler systems can be classified as *permanent, semi-permanent, portable, and continuous move. Permanent* systems have permanently located main and lateral pipelines and pumping plant. *Semi-permanent* systems consist of a permanent pumping plant and main pipeline to which portable lateral pipelines can be attached. *Portable* sprinkler systems have both portable main and lateral pipelines. *Continuous move* systems include central pivot systems and linear move systems.

Central pivot systems consist of a continuously moving lateral pipeline that rotates about a pivot point producing a circular irrigation pattern. A central pivot sprinkler system is shown below.

Figure 65. Central pivot irrigation system near Grace, Idaho.

Sprinklers can be classified as low-pressure, medium-pressure, and high-pressure sprinklers. Specific characteristics for commercial sprinkler heads, such as the pressure and discharge ranges for their operation, and the wetted diameter (effective distance of water spraying) are typically provided by manufacturers. Sprinkler types that operate under higher operating pressure provide a larger wetted diameter.

Low-pressure spray types of sprinklers include fixed sprays, spinners, and rotating sprays. Low-pressure sprinklers have become the most common sprinkler type today.

A high-pressure sprinkler generally consists of one or two nozzles that rotate under the effect of a hammer blade. The water spray impinges on a hammer blade which produces an intermittent water spray and rotates the sprinkler head. A high-pressure sprinkler head is typically mounted on a 1-inch (25 mm) diameter riser attached to a pipe.

Figure 66 shows an schematic for the layout of a sprinkler system with a pump (P), a main line, and two lateral lines. Basic information needed for the design of a sprinkler irrigation system includes a contour map of the plot to irrigate (this provides information on the ground slopes on which the pipelines will be laid), the soil characteristics (maximum application rate should not exceed the soil infiltration rate), source and quality of water available, crops to be irrigated, and local climate.

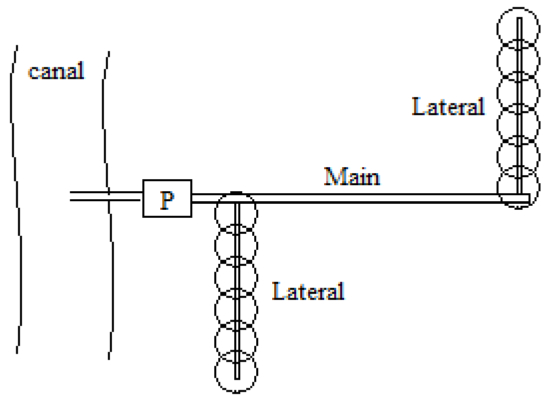


Figure 66. Schematic of a sprinkler system layout.

Some basic guidelines for sprinkler system alignment include: (1) the laterals should be perpendicular to the prevailing wind direction; (2) the main line should be as short as possible to reduce head losses; (3) the pump should be located at the center of the irrigated area if possible; and (4) provide for future expansion of the system.

Detailed design guidelines and calculations for sprinkler irrigation systems are available in NRCS, NEH15, Irrigation, Chapter 11, Sprinkle Irrigation. This reference may be downloaded from the engineering handbooks listed at:

http://directives.sc.egov.usda.gov/

0338 Microirrigation

Microirrigation (MI) is accomplished by the frequent application of small quantities of water as drops, tiny streams, or miniature spray through emitters or applicators placed along a water delivery line. MI encompasses a number of methods or concepts such as bubbler, drip, subsurface drip, mist and spray. The emitters are located at or near the plant root zone thus placing water only where the plant can use it. Thus, microirrigation can be very effective for widely spaced crops (such as orchards, melons, cucumbers).

Microirrigation systems include a control unit or system through which water is controlled, filtered, and possibly provided with additives. The control unit is typically located at the highest place in the field, and the pipelines laid parallel to the terrain slope. As with sprinkler irrigation systems, the main line in a microirrigation system is often divided into secondary branches which connect to the lines with the emitters. The main and secondary lines are frequently fitted with flow or pressure regulators. Devices are usually provided at the end of the lines to flush and clean the system.

Water is dissipated from a pipe distribution network under low pressure in a predetermined pattern. The shape and design of the emitter reduces the operating pressure in the supply line, and a small volume of water is discharged at the emission point.

Detailed design guidelines and calculations for microirrigation systems are available in NRCS NEH15, Irrigation, Chapter 7 - Trickle Irrigation. This reference may be downloaded from the engineering handbooks section of NRCS eDirectives Electronic Directives System:

http://directives.sc.egov.usda.gov/

0340 Water Flow Measurements

Water flow measurement devices and techniques for measuring pipe flow and open channel flow will be presented in this section. For additional details, refer to the USBR/USDA-NRCS/USDA-ARS (2001) *Water Measurement Manual*, available online at:

http://www.usbr.gov/pmts/hydraulics_lab/pubs/wmm/

0341 Measurements in Pipelines

Most flow measurements in pipelines are accomplished by relating the pressure drop through a device to the discharge in the pipeline.

0341. 1 Orifice Meters

An orifice meter consists of a reduction of the diameter of the flow by means of a ring inserted between two sections of a pipe. A schematic of an orifice meter is shown below.

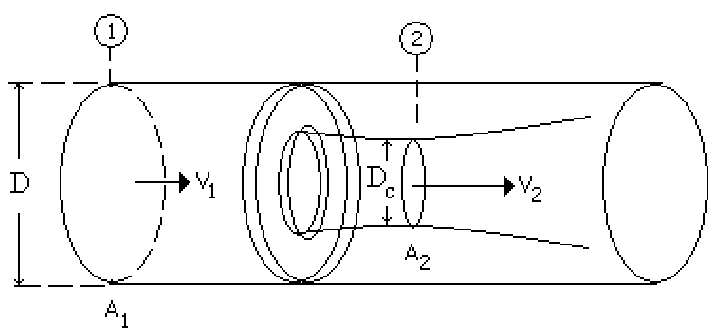


Figure 67. Schematic of flow through an orifice plate in a pipe.

Downstream of the orifice, due to the curvature of the streamlines detaching from the orifice, the flow cross-section contracts such that the area at this *vena contracta* is smaller than the area of the orifice. Let A_o be the area of the orifice, then, the area of the vena contracta can be written as $A_2 = C_c A_o$, where C_c (a value less than 1.0) is known as the *contraction coefficient*.

Assuming that the contraction of the flow does not occur, an ideal velocity at section 2, say, V_i , could be calculated in terms of continuity, i.e., $V_1A_1 = V_iA_o$. The actual velocity at the vena contracta can be calculated by $V_2 = C_vV_i$, where C_v is a velocity coefficient.

The theoretical discharge would be $Q_i = V_iA_o$, however, the real discharge is calculated as $Q = V_2A_2 = (C_vV_i)(C_cA_o) = (C_cC_v)(V_iA_o) = C_dQ_i$. Thus, we have introduced a discharge coefficient $C_d = C_cC_v$ to calculate the actual discharge in terms of the theoretical discharge.

Figure 68, below, shows typical values of the contraction (C_c), velocity (C_v), and discharge (C_d) coefficients for different orifices.

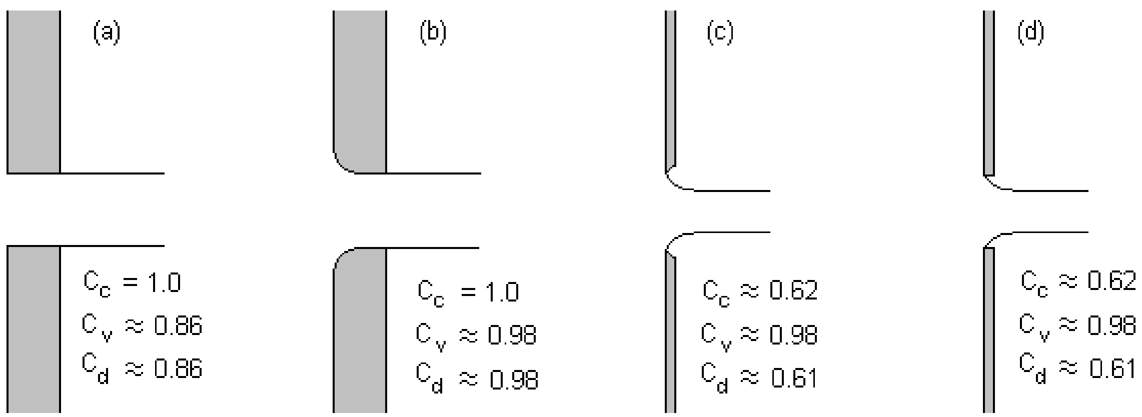


Figure 68. Contraction, velocity, and discharge coefficients for flow through orifices: (a) sharp thick wall, (b) rounded thick wall, (c) chiseled thin wall, (d) sharp thin wall.

To relate the discharge through the orifice to the drop in pressure between sections (1) and (2) in a horizontal pipeline, such as in Figure 67, the Bernoulli equation is used. Including a discharge coefficient, the discharge through the orifice is given by

$$Q = C_d A_1 A_2 \sqrt{\frac{2g(h_1 - h_2)}{A_1^2 - A_2^2}} \qquad \textbf{[Eq. 206]}$$

Where h_1 and h_2 are the piezometric heads at sections (1) and (2), respectively, with $h_1 = z_1 + p_1/\omega$, and $h_2 = z_2 + p_2/\omega$. If the orifice meter is located in a horizontal pipeline, the piezometric heads may be written as $h_1 = p_1/\omega$, and $h_2 = p_2/\omega$. The piezometric heads are illustrated in the following figure.

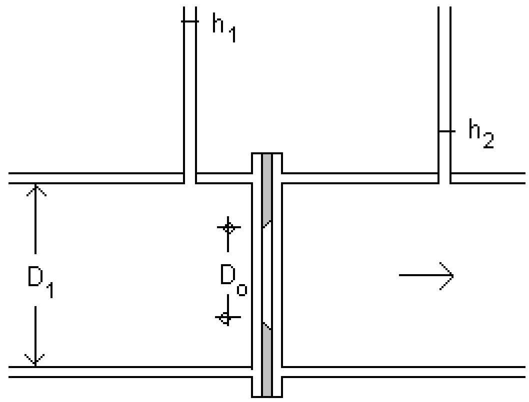


Figure 69. Schematic of orifice plate

Figure 70. Plexiglas pipe with an orifice meter and piezometric tubes, used in the laboratory to demonstrate orifice flow

Example 87 – Flow discharge through an orifice meter

An orifice meter of diameter $D_o = 0.30\ ft$ is placed in a pipeline of diameter $D_1 = 0.50\ ft$. Measurements of the piezometric heads upstream and downstream of the orifice indicate that $h_1 = 2.0\ ft$ and $h_2 = 1.3\ ft$. Using a chiseled thin-walled orifice with a contraction coefficient $C_c = 0.62$ and a discharge coefficient $C_d = 0.61$, determine the flow through the pipe.

The areas of the pipe and orifice are: $A_1 = \pi D_1^2/4 = \pi(0.50)^2/4 = 0.196\ ft^2$, $A_o = \pi D_o^2/4 = \pi(0.30)^2/4 = 0.070\ ft^2$. The area of the vena contracta is $A_2 = C_c A_o = 0.62 \times 0.070 = 0.043\ ft^2$. The discharge is:

$$Q = C_d A_1 A_2 \sqrt{\frac{2g(h_1 - h_2)}{A_1^2 - A_2^2}} = 0.61 \times 0.196 \times 0.043 \times \sqrt{\frac{2 \times 32.2 \times (2.0 - 1.3)}{0.196^2 - 0.043^2}} = 0.18\, cfs$$

0341.2 Venturi Meters

A Venturi meter consists of a contracted section (known as the *throat*) connected to the pipe upstream through a short contraction, and to the pipe downstream through a longer expansion. The diameter of the throat is D_2, while that of the pipe is D_1.

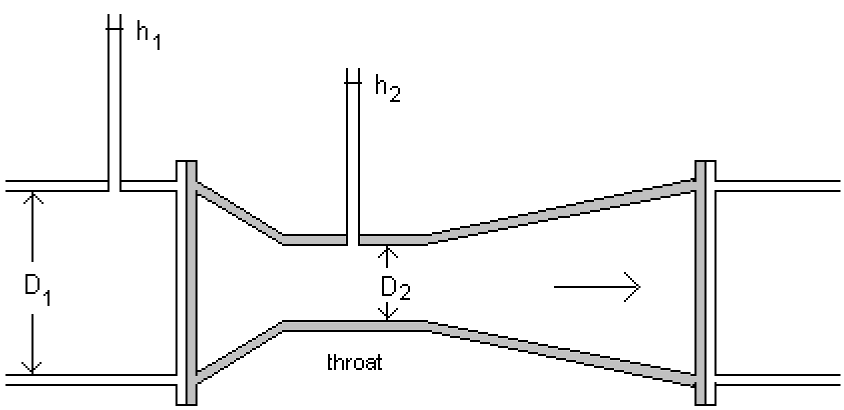


Figure 71. Schematic of a Venturi meter

The equation for the discharge in the Venturi meter is the same as for the orifice meter, except that A_2 corresponds to the throat area and is not calculated through a contraction coefficient. The discharge coefficient C_d can be found from calibration.

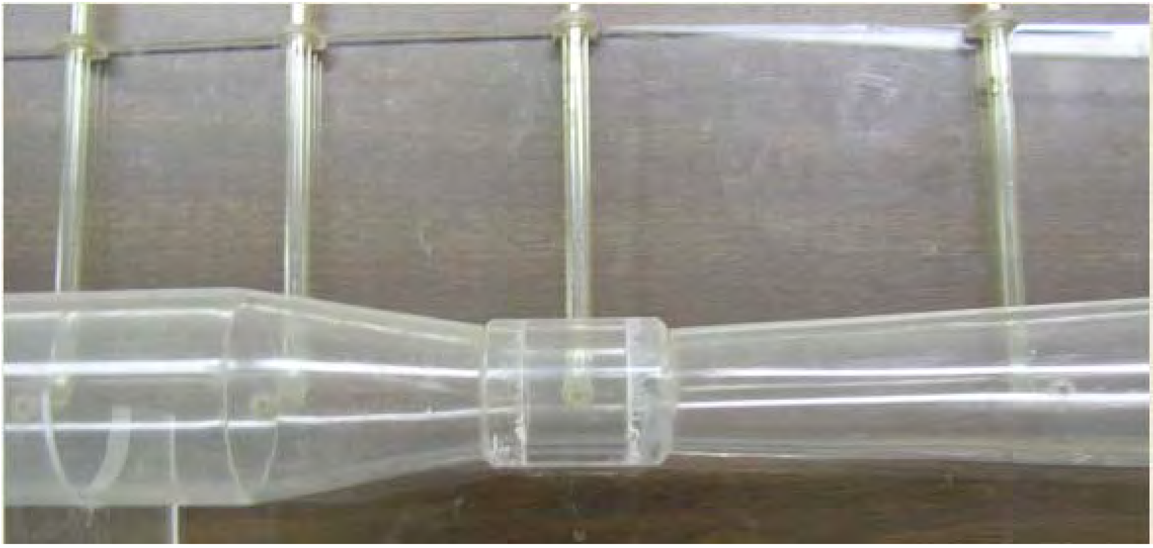


Figure 72. Plexiglas Venturi meter with piezometric tubes, to demonstrate flow through the meter.

Figure 73. Venturi meter installed in a 4-inch pipeline, with manometer lines attached at the upstream end and at the throat of the meter.

Example 88 – Flow discharge through a Venturi meter

A Venturi meter with a throat diameter $D_2 = 0.5\ ft$ is installed in a pipe of diameter $D_1 = 1.0\ ft$. Measurements of the piezometric heads upstream and at the throat of the Venturi meter indicate that $h_1 = 2.5\ ft$ and $h_2 = 0.5\ ft$. With a discharge coefficient $C_d = 0.8$, determine the flow through the pipe.

The areas of the pipe and of the Venturi's throat are:

$$A_1 = \pi D_1^2/4 = \pi(1.00)^2/4 = 0.785\ ft^2,$$

$$A_2 = \pi D_2^2/4 = \pi(0.50)^2/4 = 0.196\ ft^2.$$

The discharge is (equation 206):

$$Q = C_d A_1 A_2 \sqrt{\frac{2g(h_1 - h_2)}{A_1^2 - A_2^2}} = 0.8 \times 0.785 \times 0.196 \times \sqrt{\frac{2 \times 32.2 \times (2.5 - 0.5)}{0.785^2 - 0.196^2}} = 1.84\, cfs\,.$$

0341.3 Nozzle Meters

Nozzle meters reduce the flow cross-section, as does the orifice meter, by utilizing a nozzle attachment illustrated below.

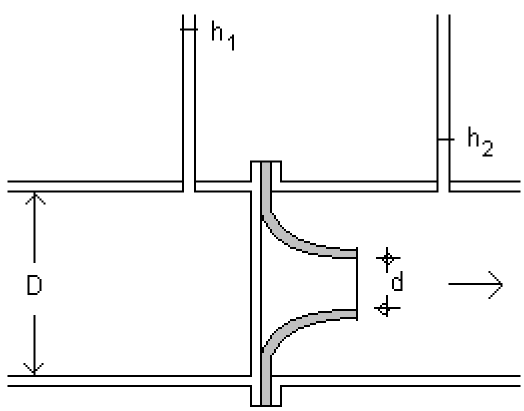


Figure 74. Schematic of a nozzle meter.

The nozzle is characterized by its opening diameter *d*, and area, A_d. The equation for the discharge is written as:

$$Q = C_d A_d \sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{d}{D}\right)^4}} \qquad \textbf{[Eq. 207]}$$

Where h_1 and h_2 are the piezometric heads at sections (1) and (2), respectively, and C_d is the discharge coefficient. The value of C_d depends on the type of nozzle, and is obtained by calibration. Calibration can be performed by attaching the nozzle to a pipe and measuring the discharge Q through the pipe for different pressure drops (h_1-h_2). Since the diameters d and D can be measured or are provided by the manufacturer, equation 207 can be used to obtain the discharge coefficient C_d.

Example 89 – Flow discharge through a nozzle meter

A nozzle meter whose opening has a diameter $d = 0.10$ *ft* is placed in a pipeline of diameter $D_1 = 0.25$ *ft*. Measurements of the piezometric heads upstream and downstream of the orifice indicate that $h_1 = 2.5$ *ft* and $h_2 = 1.5$ *ft*. With a discharge coefficient $C_d = 0.90$, determine the flow through the pipe.

The area of the nozzle opening is:

$$A_d = \pi d^2/4 = \pi(0.10)^2/4 = 0.0078\ ft^2$$

The discharge is

$$Q = C_d A_d \sqrt{\frac{2g(h_1 - h_2)}{1 - \left(\frac{d}{D}\right)^4}} = 0.90 \times 0.0078 \times \sqrt{\frac{2 \times 32.2 \times (2.5 - 1.5)}{1 - \left(\frac{0.10}{0.25}\right)^4}} = 0.057\, cfs$$

0341.4 Elbow Meters

An elbow meter consists of attaching pressure transducers to the inner and outer walls of a pipe elbow, as shown in Figure 75 below.

Let $\Delta p = |p_1 - p_2|$ be the absolute value of the pressure difference between the inner and outer walls of the elbow. Equivalently, the pressure difference can be written as $\Delta p = \omega \cdot \Delta h$, where ω is the specific weight of the liquid. The discharge through the elbow can be calculated as

$$Q = C\sqrt{\frac{\Delta p}{\omega}} = C\sqrt{\Delta h} \qquad \textbf{[Eq. 208]}$$

where C is a calibration constant unique for a given elbow. If the pressure taps in the elbow meter were connected to a manometer, the quantity Δh could represent the reading in the manometer. Refer to section *0311.1 Piezometers and Manometers* for detailed explanation of manometers.

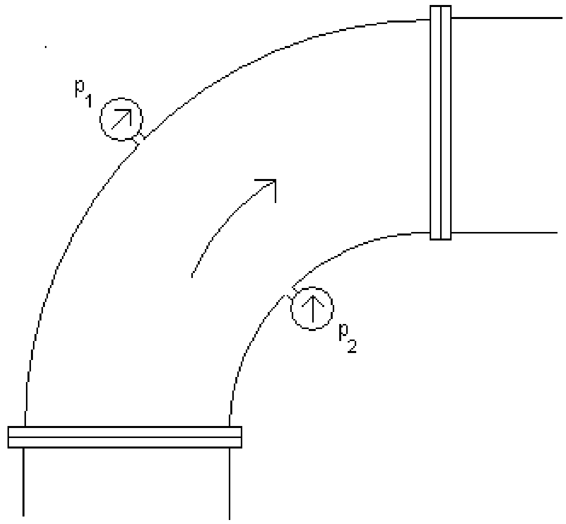


Figure 75. Schematic of an elbow meter.

Example 90 – Flow discharge through an elbow meter

An elbow meter has been calibrated so that when the pressure difference between its inner and outer walls is measured in *ft*; the calibration constant is *C = 15.2*. Determine the discharge, in *cubic feet per second* (*cfs*), when *Δh = 0.5 ft*.

The discharge is: $Q = C\sqrt{\Delta h} = 15.2 \times \sqrt{0.5} = 10.75\,cfs$

For the orifice, nozzle, Venturi, and elbow meters, the discharge *Q* varies with the square root of the piezometric head drop *Δh*. A plot of this relationship is referred to as the *rating curve* (of characteristic shape). Note that cross-section rating curves in open channel flow have the same characteristic shape, but with the axes exchanged. The elbow meter used in the example above has a rating curve shown in the figure below.

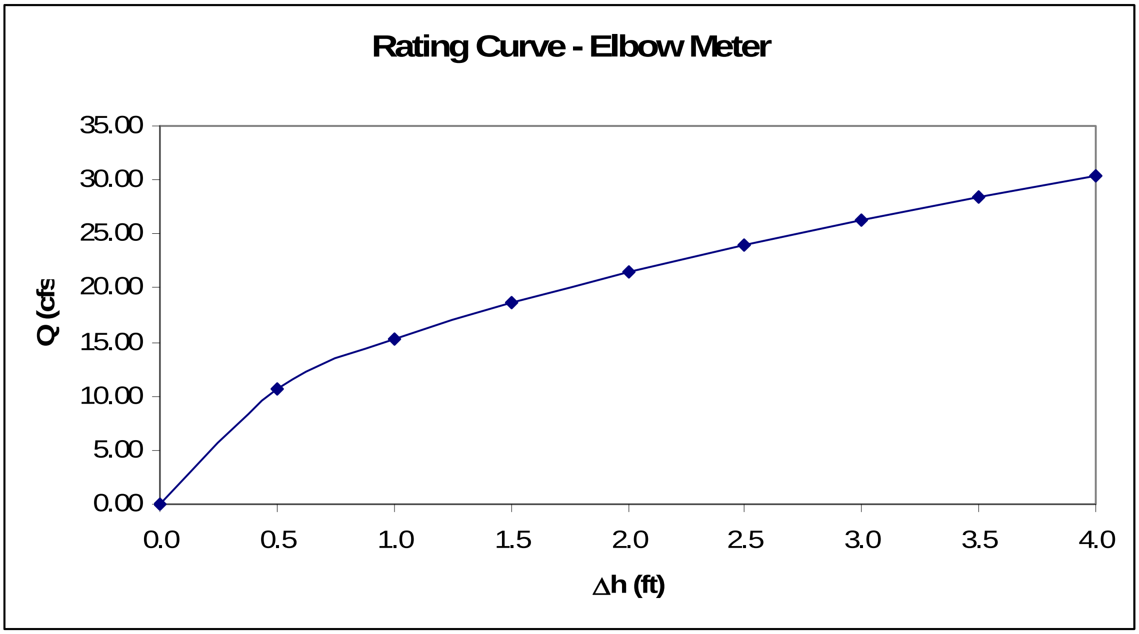


Figure 76. Values of the discharge can be interpolated graphically from the rating curve of a given device.

0341.5 Magnetic and Ultrasonic Meters

Magnetic meters are attached to the outside of a pipe or sometimes placed inside the pipe to measure discharge. In a magnetic meter the distortion in the magnetic field due to the moving water is related to the flow discharge. Magnetic meters are calibrated so that a voltage read in the meter can be converted to discharge. There are a variety of commercial magnetic meters. Instructions on the installation and operation of magnetic meters are available from the manufacturers.

The ultrasonic flowmeter is a clamp-on device that measures pipeline flow using the principle of sonic beam phase shift. Instructions on the installation and operation of ultrasonic meters are available from the manufacturers. Research information may also be available from university extension services such as the March 2008 *Using Ultrasonic Flow Meters in Irrigation Applications* from the University of Nebraska-Lincoln Extension, Institute of Agriculture and Natural Resources:

http://www.ianrpubs.unl.edu/epublic/live/g1426/build/g1426.pdf

0342 Measurements in Open Channels

Measurements in open channel flow include depth, velocity, and discharge.

0342.1 Depth Measurements

Depth measurements can be carried out by using a graduated rod or ruler, whether permanent or portable. The figure below shows a permanent depth gage in an open channel.

Figure 77. Depth gage showing water surface elevation.

In small laboratory flumes, point gages with an attached Vernier scale can be used to measure the water depth. When using a point gage in a flume, the Vernier scale is read when the point gage touches the flume bed, and then when the point gage touches the surface of the water. The difference between these two readings is the water depth. Figure 78, below, shows a point gage with a Vernier scale.

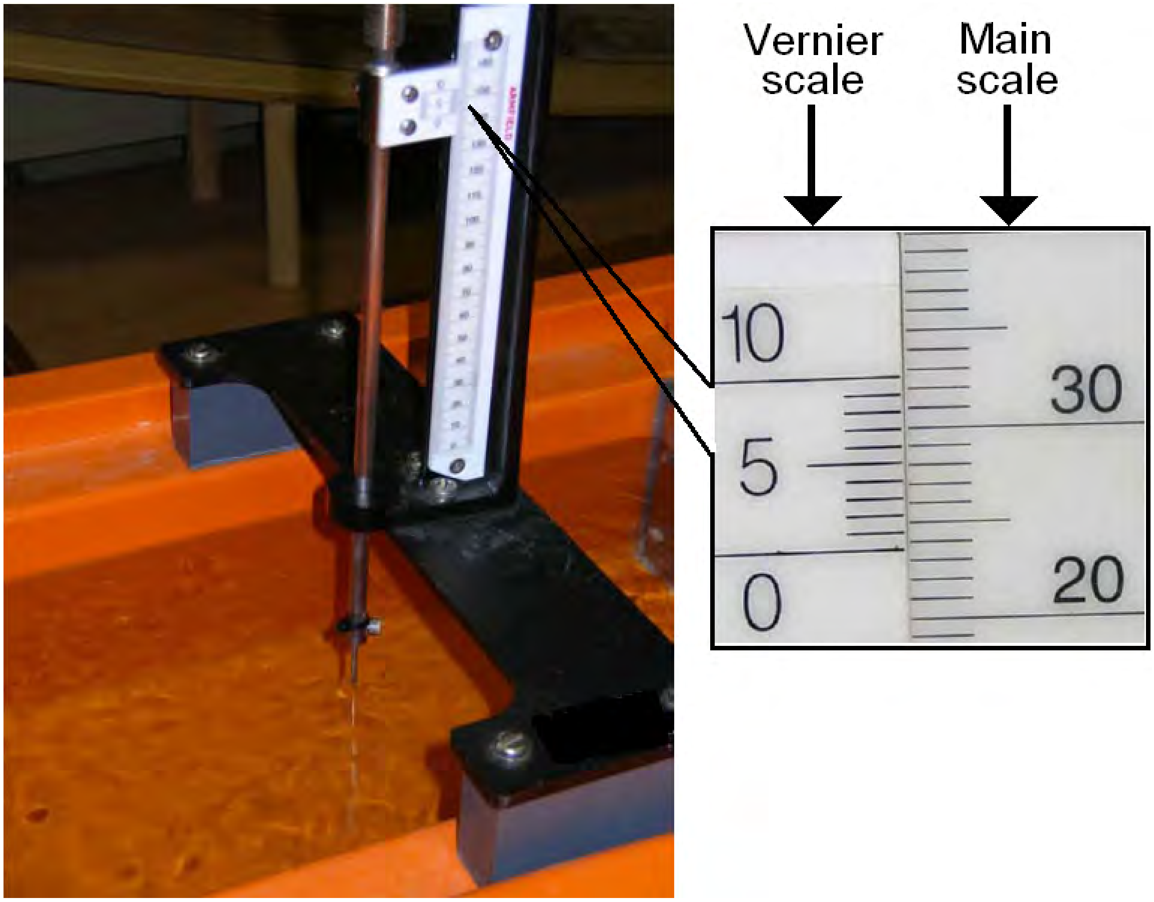


Figure 78. Point gage in laboratory flume with Vernier scale reading of *23.7*.

0342.2 Velocity Measurements

Measurements of the water velocity in open channel flow can be performed through a variety of methods. Some of these methods are discussed in the following sections.

0342.2.1. Propeller/Paddle Wheel Meters

A propeller or paddle wheel meter consists of a moving propeller or buckets whose rate of rotation can be recorded and related to the local flow velocity. Calibration information for these meters can be obtained from the manufacturer or from laboratory tests. Small propeller meters can be used in laboratory flumes to measure local velocities. Propeller meters used in larger streams typically have a streamlined body attached to the propeller to minimize effects on the flow.

0342.2.2. Vortex Meter

Obstacles in a flow, such as a cylinder or a block, generate a train of vortices downstream of the obstacles. A vortex flow meter detects the vortex field produced by a given obstacle and relates it to the local velocity. Vortex flow meters are available from a number of manufacturers. Information on setting up and operating these meters can be obtained from the manufacturers.

0342.2.3. Doppler (Acoustic) Meters

Doppler (or acoustic) meters use an acoustic signal released into the water at a point and picked up by a sensor a short distance downstream. The time required to pick up the signal is related to the local flow velocity. A variety of commercial Doppler meters exists. Details for the set up and operation of these meters are available from the manufacturers.

0342.2.4 Velocity Measurements with Floaters

A simple way to measure velocity in open channels consists of measuring the time, t, required by a floater to travel a certain distance, x, in the channel. A straight channel reach is needed to perform floater velocity measurements. Neutrally-buoyant floaters are preferred because they are less impacted by wind. The flow velocity at the free surface is simply, x/t. Measurements in laboratory flumes indicate that the mean velocity is *0.85* times the free-surface velocity.

Example 91 – Flow velocity calculation using the float method

While performing a floater velocity measurement in a straight reach of an irrigation canal it is found that the floater requires *56 sec* to travel *100 ft*. Estimate the surface velocity as well as the mean velocity in the canal.

With $x = 100\ ft$, and $t = 56\ sec$, the surface velocity is $V_s = x/t = 100ft/56\ sec = 1.78\ fps$. The mean velocity is $V = 0.85V_s = 0.85\times1.78\ fps = 1.52\ fps$.

0342.2.5. Laser-Doppler and Particle-Velocity Measurements

These types of velocity measurements are suitable in the laboratory. In a Laser-Doppler velocimeter, two or three laser beams are focused at a point in the flow. Air bubbles or any other impurity in the water passing through the laser focus point will produce a Doppler shift which is detected by an optical sensor. Calibration of the laser Doppler velocimeter allows for the determination of the local velocity at the laser focus point.

In a Particle velocimeter, tracers introduced in the flow reflect light from a source and the location of the particles is optically traced allowing for the instantaneous description of flow fields.

0342.3 Sharp-crested Weirs

Sharp-crested weirs may be as simple as a thin plate placed across a rectangular open channel, as illustrated in Figure 79 below. Such a weir is referred to as a *suppressed weir*, as opposed to a *contracted weir* shown in Figure 80. The difference is that in a contracted weir the weir crest, length = L, does not extend the full width of the channel, b. Whereas in a suppressed weir $b = L$. In both cases of rectangular weirs P represents the weir height and H is the weir head. The head H is measured from the crest of the weir to the water surface at a point upstream so that the curvature of the flow streamlines is minimal (3 to 4 times the maximum value of H expected is suggested), as illustrated below.

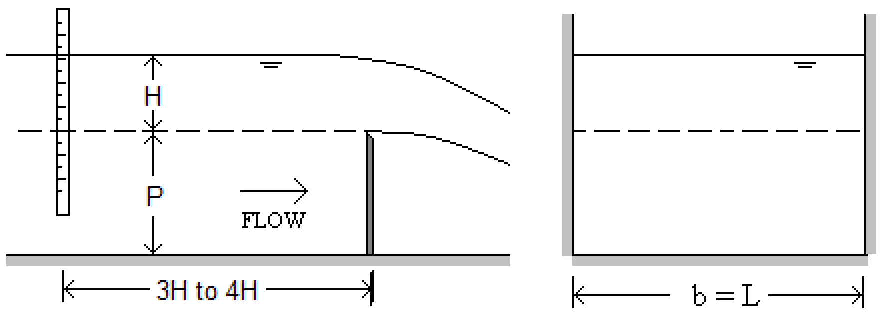


Figure 79. Suppressed sharp-crested weir in a rectangular channel.

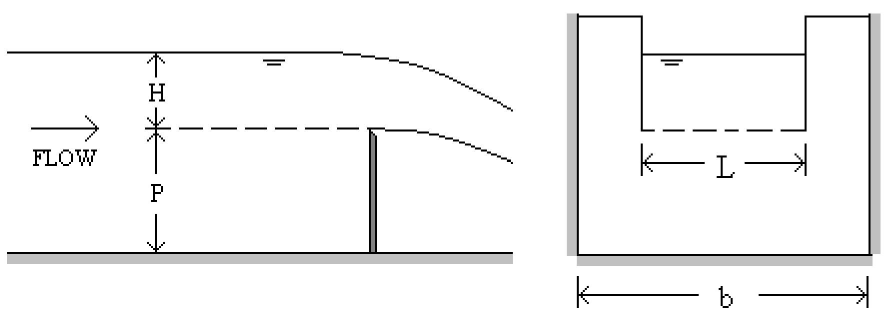


Figure 80. Contracted sharp-crested weir in a rectangular channel.

For the suppressed sharp-crested weir, the discharge Q over the weir is calculated using:

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot L \cdot H^{3/2} \qquad \textbf{[Eq. 209]}$$

Where C_d is a discharge coefficient, g is the acceleration of gravity ($g = 9.806\ m/s^2 = 32.2\ ft/s^2$). The quantities L and H were defined previously. The value of C_d can be obtained using Rehbock's formulas:

$$C_d = 0.605 + \frac{1}{305 \cdot H} + 0.08 \cdot \frac{H}{P},\ H(\text{ft}),\ P(\text{ft}) \qquad \textbf{[Eq. 210]}$$

or,

$$C_d = 0.605 + \frac{1}{1000 \cdot H} + 0.08 \cdot \frac{H}{P},\ H(\text{m}),\ P(\text{m}) \qquad \textbf{[Eq. 211]}$$

These equations are valid for *P* in the range of *0.33 ft* to *3.3 ft* (*0.10 m* to *1.0 m*) and for *H* in the range *0.08 ft* to *2.0 ft* (*0.025 m* to *0.60 m*), and the ratio $H/P < 1.0$.

A photograph of a sharp-crested weir is shown below.

Figure 81. Sharp-crested weir in operation (USDA).

Example 92 – Discharge over a suppressed rectangular weir

A suppressed rectangular weir is located in a *3-ft* wide rectangular open channel. If the weir is *0.5 ft* high and the head measured upstream of the weir is $H = 0.4$ *ft*, determine the discharge over the weir using Rehbock's formulas.

For this case $L = b = 3$ *ft*, $P = 0.5$ *ft*, and $H = 0.4$ *ft*, and $H/P = 0.4/0.5 = 0.8$. Rehbock's formula in units of the English System indicates that the discharge coefficient C_d is given by:

$$C_d = 0.605 + \frac{1}{305 \cdot H} + 0.08 \cdot \frac{H}{P} = 0.605 + \frac{1}{305 \times 0.4} + 0.08 \times 0.8 = 0.677 .$$

The discharge is calculated as:

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot L \cdot H^{3/2} = 0.677 \times \frac{2}{3} \times \sqrt{2 \times 32.2\, ft/s^2} \times 3\, ft \times (0.4\, ft)^{3/2} = 2.75\, cfs .$$

In equation 209, a *weir coefficient* C_w can be defined as $C_w = C_d \cdot \frac{2}{3} \cdot \sqrt{2g}$. And for $H/P < 0.4$, the discharge coefficient is approximately $C_d = 0.62$; thus, the equation can be re-written as:

$$Q = 3.32 \cdot L \cdot H^{3/2} \text{, } Q(cfs), L(ft), H(ft) \qquad \textbf{[Eq. 212]}$$

or

$$Q = 1.83 \cdot L \cdot H^{3/2} \text{, } Q(m^3/s), L(m), H(m) \qquad \textbf{[Eq. 213]}$$

Example 93 – Discharge over a suppressed rectangular weir
A suppressed rectangular weir is located in a *4-ft* wide rectangular open channel. If the weir is *1.0 ft* high and the head measured upstream of the weir is $H = 0.25\ ft$, determine the discharge over the weir using the formula for $H/P<0.4$.

$$Q = 3.32 \cdot L \cdot H^{3/2} = 3.32 \times 4.0 \times (0.25)^{3/2} = 1.66\, cfs$$

For a contracted sharp-crested weir, the equation to calculate the discharge is the *Francis* formula:

$$Q = C_w \cdot (L - 0.1 \cdot n \cdot H) \cdot H^{3/2} \quad \textbf{[Eq. 214]}$$

The value of n is either "1" or "2", depending on whether the contractions occur on one or two sides, as illustrated below.

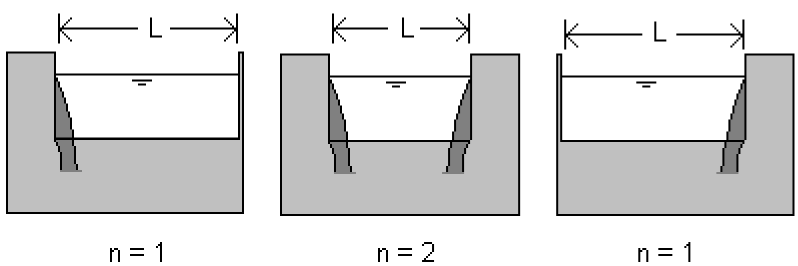


Figure 82. Contracted rectangular weir with *n = 1* or *n = 2* contractions.

Example 94 – Discharge over a contracted weir
A contracted weir of length $L = 2\ ft$ is set in a *3-ft* wide rectangular open channel. Using a weir coefficient $C_w = 3.32$, determine the discharge for a head $H = 0.20\ ft$ with (a) $n = 1$ contraction, and (b) $n = 2$ contractions. The weir height is $P = 0.5\ ft$.

The ratio $H/P = 0.20ft/0.5ft = 0.4$, and the discharge is

(a) $Q = C_w \cdot (L - 0.1 \cdot n \cdot H) \cdot H^{3/2} = 3.32 \times (2 - 0.1 \times 1 \times 0.2) \times 0.2^{3/2} = 0.588\, cfs$

(b) $Q = C_w \cdot (L - 0.1 \cdot n \cdot H) \cdot H^{3/2} = 3.32 \times (2 - 0.1 \times 2 \times 0.2) \times 0.2^{3/2} = 0.582\, cfs$

Figure 83 below shows a contracted rectangular weir in a laboratory flume.

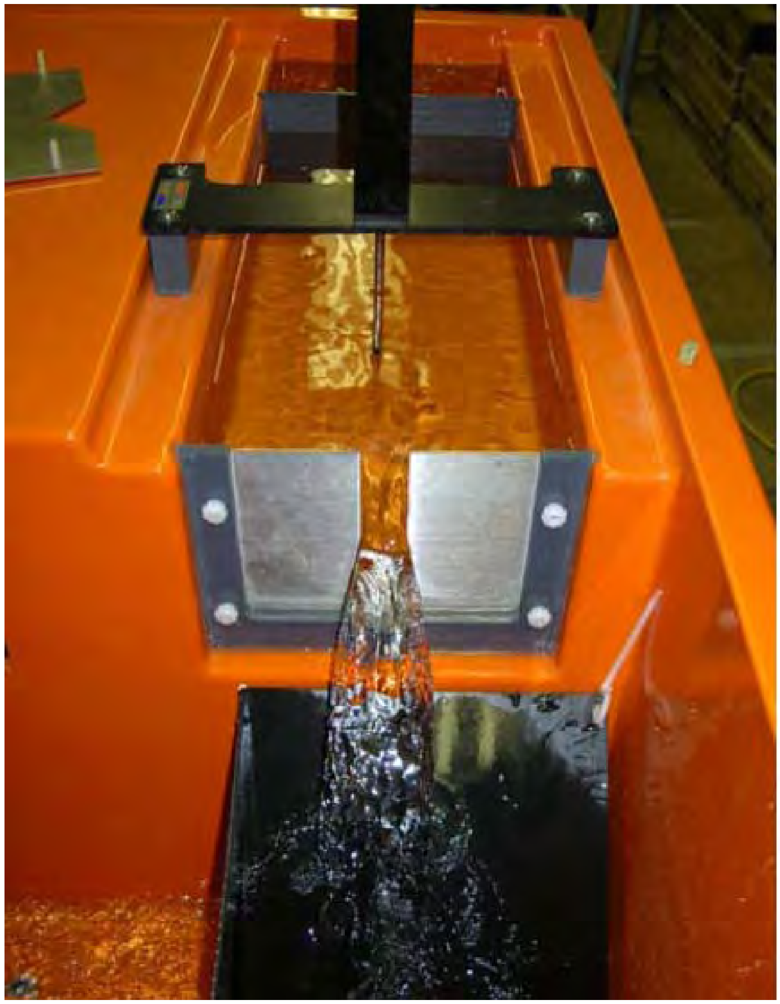


Figure 83. Contracted weir in a laboratory flume.

A special type of a contracted weir is the *Cipoletti weir* consisting of a trapezoidal shape with side slopes 1H: 4V as shown below.

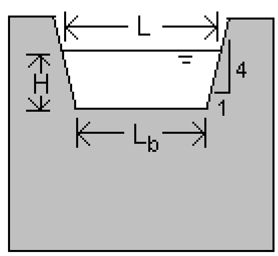


Figure 84. Cipoletti weir schematic.

The equation for the discharge over a Cipoletti weir is given by:

$$Q = 3.367 \cdot L \cdot H^{3/2}, Q(cfs), L(ft), H(ft) \qquad \textbf{[Eq. 215]}$$

Where $L = L_b + H/2$, and L_b is the bottom or crest length of the Cipoletti weir.

Example 95 – Discharge over a Cipoletti weir

Determine the discharge over a Cipoletti weir if the weir length is $L = 1.5\ ft$, and the head is $H = 0.5\ ft$.

The discharge is $Q = 3.367 \cdot L \cdot H^{3/2} = 3.367 \times 1.5 \times 0.5^{3/2} = 1.79\, cfs.$

A triangular, or v-notch weir consists of a symmetric v-shaped notch in a thin wall placed across an open channel. A v-notch weir is used to measure low flows of less than 10 cfs. A schematic of a triangular (or v-notch) weir is shown below.

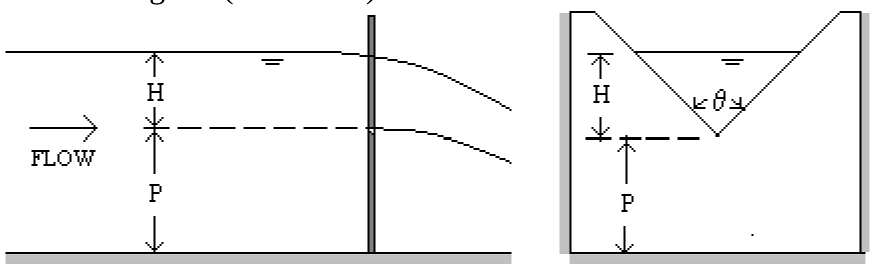


Figure 85. Schematic of a triangular or v-notch weir.

. Figure 86 below shows a small triangular weir in a laboratory flume.

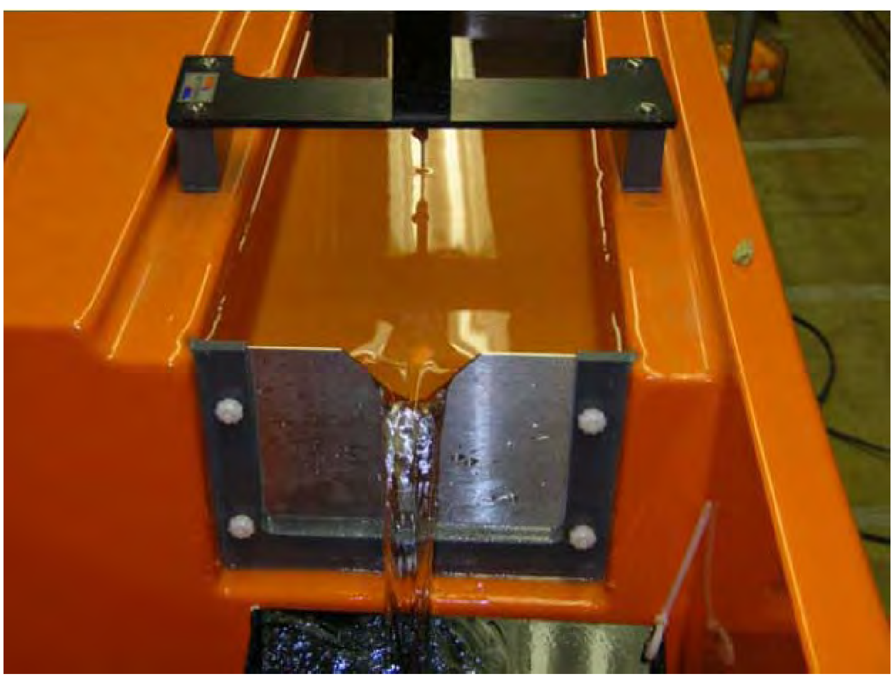


Figure 86. V-notch, or triangular, weir in a laboratory flume.

The discharge for a triangular (or v-notch) weir is given by:

$$Q = C_d \cdot \frac{8}{15} \cdot \sqrt{2g} \cdot \tan\left(\frac{\theta}{2}\right) \cdot H^{5/2} \qquad \textbf{[Eq. 216]}$$

Where C_d is a discharge coefficient, g is the acceleration of gravity ($g = 9.806\ m/s^2 = 32.2\ ft/s^2$), and θ is the angle of the v-notch. A typical value of the contraction coefficient is $C_d = 0.58$.

Example 96 – Discharge over a triangular weir

Determine the discharge over a v-notch weir if the weir angle is $\theta = 80^o$ and the head is $H = 0.55\ ft$. Use a discharge coefficient $C_d = 0.58$.

The discharge is:

$$Q = C_d \cdot \frac{8}{15} \cdot \sqrt{2g} \cdot \tan\left(\frac{\theta}{2}\right) \cdot H^{5/2} = 0.58 \times \frac{8}{15} \times \sqrt{2 \times 32.2\ ft/s^2} \times \tan\left(\frac{80^o}{2}\right) \times (0.55)^{5/2}$$

$$Q = 0.467\, cfs$$

0342.4 Broad-crested Weirs

A broad-crested weir consists of an obstacle of height P placed in a rectangular open channel, as illustrated below. The flow over the broad crested weir becomes critical, so that the depth at that point is the critical depth y_c.

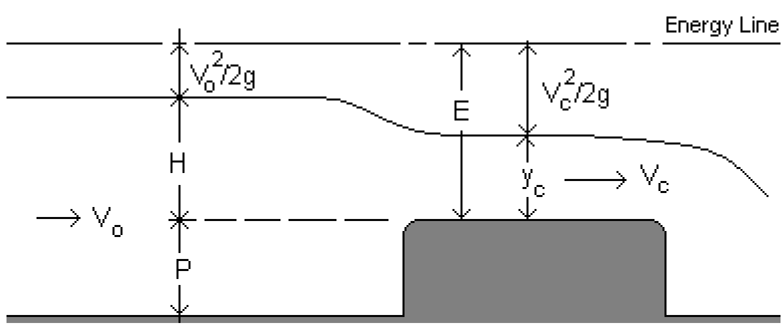


Figure 87. Schematic of flow over a broad-crested weir.

In a rectangular channel of width L, the discharge over the broad-crested weir is:

$$Q = L\sqrt{g \cdot y_c^3} \qquad \textbf{[Eq. 217]}$$

If E represents the specific energy available over the weir, the critical depth can be expressed as $y_c = \frac{2}{3}E$, and the discharge is written as:

$$Q = L\sqrt{g} \cdot y_c^{3/2} = L\sqrt{g}\left(\frac{2}{3}\right)^{3/2} E^{3/2} \qquad \textbf{[Eq. 218]}$$

Expressing this equation in the form of a suppressed weir equation (equation 209), the discharge coefficient is given by:

$$C_d = \frac{1}{\sqrt{3}}\left(\frac{E}{H}\right)^{3/2} \qquad \textbf{[Eq. 219]}$$

Where H is the head measured upstream of the broad-crested weir, where the flow depth is $y = P+H$ and the flow velocity is V_o. For a large value of H, V_o is small, and E becomes similar to H, i.e., E/H becomes 1.0 which provides a minimum value for the discharge coefficient, $(C_d)_{min} = \frac{1}{\sqrt{3}} = 0.577$. As the velocity V_o increases, the value of E/H becomes larger than 1, and the discharge coefficient (equation 219) becomes larger than 0.577.

If the critical depth can be measured over the weir, equation 217 provides the discharge. If the head H is measured use equation 209, instead, with a discharge coefficient larger than or equal to 0.577. These are the equations used to evaluate many of the long-throated flume types discussed in section 0342.6.1.

Example 97– Discharge over a broad-crested weir- critical depth measured

Determine the discharge over a broad-crested weir if the critical depth of flow over the weir is measured to be $y_c = 0.52\ ft$. The channel has a width $L = 2.5\ ft$.

The discharge is:

$$Q = L\sqrt{g \cdot y_c^3} = 2.5\ ft \times \sqrt{32.2\ ft/s^2 \times (0.52\ ft)^3} = 5.32\, cfs$$

Example 98 – Discharge over a broad-crested weir – head measured

Determine the discharge over a broad-crested weir if the head measured upstream of the weir is measured to be $H = 0.65\ ft$. The channel has a width $L = 3.5\ ft$. Use a discharge coefficient $C_d = 0.61$.

The discharge is (equation 209):

$$Q = C_d \cdot \frac{2}{3} \cdot \sqrt{2g} \cdot L \cdot H^{3/2} = 0.61 \times \frac{2}{3} \times \sqrt{2 \times 32.2\, ft/s^2} \times 3.5\, ft \times (0.65\, ft)^{3/2} = 5.99\, cfs$$

Most hydraulic structures in natural resource conservation work incorporate broad-crested weirs, where their function is the control of flood flows. The crest is horizontal and long in the direction of flow so that the water lays on the crest rather than springing free as water does flowing over a sharp-crested weir. Roadways over bridges and culverts may be considered broad-crested weirs to estimate overtopping flows. These discharges are generally calculated with equation 209, where a weir coefficient, C_w, can be defined as $C_w = C_d \cdot \frac{2}{3} \cdot \sqrt{2g}$. C_w ranges in value from 2.5 to 3.1.

0342.5 Submerged Weir Flow

Conditions downstream of a sharp-crested weir may produce submergence of the weir as illustrated. The equation used to calculate the discharge is the same as equation 212, modified by a submergence coefficient C_s, and replacing H with the upstream head H_u. The resulting equation is:

$$Q = 3.32 \cdot C_s \cdot L \cdot H_u^{3/2} \qquad \textbf{[Eq. 220]}$$

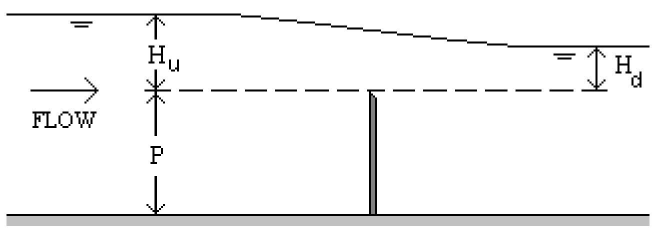


Figure 88. Submerged weir flow.

The submergence coefficient is based on empirical data, as shown in Figure 89. A polynomial fitting of this empirical data gives the following equation for the coefficient of submergence as:

$$C_s = -28.152 \cdot \left(\frac{H_d}{H_u}\right)^4 + 62.59 \cdot \left(\frac{H_d}{H_u}\right)^3 - 51.395 \cdot \left(\frac{H_d}{H_u}\right)^2 + 18.165 \cdot \left(\frac{H_d}{H_u}\right) - 1.3319$$

[Eq. 221]

for $0.38 < H_d/H_u < 1.0$, or $C_s = 1.0$, for $H_d/H_u < 0.38$.

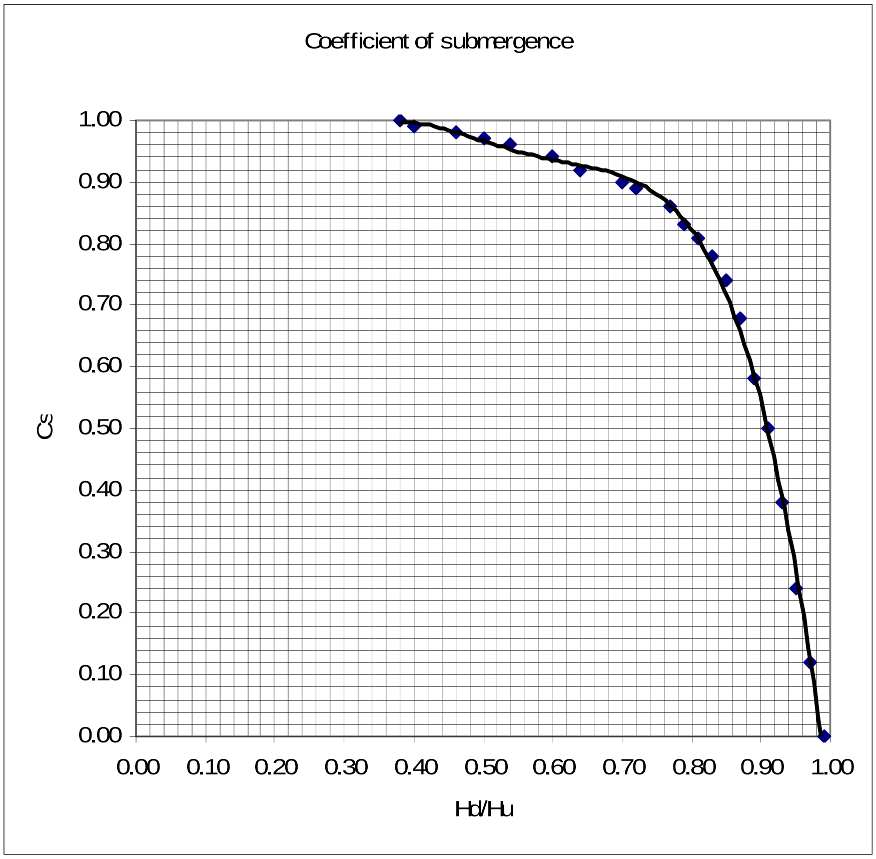


Figure 89. Coefficient of submergence for sharp crested weirs.

Example 99 - Discharge over a submerged sharp-crested weir

A sharp crested weir is operating under submerged conditions with upstream and downstream heads of $H_u = 0.40\ ft$ and $H_d = 0.25\ ft$. If the weir has a length $L = 3.5\ ft$, determine the discharge Q over the weir.

The submergence ratio $H_d/H_u = 0.25\ ft/0.40\ ft = 0.625$, and the submergence coefficient is calculated as:

$$C_s = -28.152 \cdot \left(\frac{H_d}{H_u}\right)^4 + 62.59 \cdot \left(\frac{H_d}{H_u}\right)^3 - 51.395 \cdot \left(\frac{H_d}{H_u}\right)^2 + 18.165 \cdot \left(\frac{H_d}{H_u}\right) - 1.3319 =$$

$$C_s = -28.152 \cdot (0.625)^4 + 62.59 \cdot (0.625)^3 - 51.395 \cdot (0.625)^2 + 18.165 \cdot (0.625) - 1.3319 =$$

$$C_s = 0.93$$

With this coefficient, the discharge is calculated as:

$$Q = 3.32 \cdot C_s \cdot L \cdot H_u^{3/2} = 3.32 \times 0.93 \times 3.5 \times 0.40^{3/2} = 2.73 cfs$$

Submergence affects broad-crested weirs less than sharp-crested weirs. For broad-crested weirs, discharge begins to be reduced at 80% submergence; at 90% submergence, discharge is still greater than 90% of unsubmerged discharge, USACOE (2008).

0342.6 Flumes

Flumes are shaped, open channel flow sections that force flow to accelerate and pass through critical depth in the flume. When flow passes through critical depth, a unique water surface profile occurs within the flume for each discharge. Flow acceleration is produced by converging the sidewalls, raising the bottom, or a combination of both. Flumes range in size from 1- inch to over 50 ft wide, and are installed in ditches, laterals, and large canals to measure flow.

0342.6.1 Long-throated Flumes

Long-throated flumes are coming into general use because they can be easily fitted into channel shapes. When the flume's bottom is a raised overflow crest, with no side contractions, the flume is commonly called a broad-crested weir. Characteristics of long-throated flumes include:

1. Can have nearly any desired cross-sectional shape,
2. Can be made into portable devices,
3. Have few problems with floating debris and sediment,
4. May be designed and calibrated by computer techniques; WinFlume design software and additional information can be downloaded at:

http://www.wsi.nrcs.usda.gov/products/W2Q/water_mgt/Irrigation/irrig-mgt-models.html

5. And, generally have measurement errors less than ±2%.

0342.6.2 Parshall Flumes

Parshall flumes are used to measure water flow by custom or by law in some locales. The Parshall flume is an open-channel measurement structure that combines a contraction of the channel width with a drop in the channel bed. Both transitions contribute to the establishment of critical flow, thus providing a unique relationship between the flow depth and discharge through the flume. Parshall flumes are calibrated empirically, using other more precise and accurate water-measuring systems. Since the flumes are calibrated empirically, care must be taken to build Parshall flumes according to the design dimensions. If h_a represents the water depth at the Parshall flume's throat in *ft*, the discharge through the flume Q in *cfs*, can be calculated using the formula:

$$Q = C\, h_a^{\,n}, \qquad \textbf{[Eq. 222]}$$

Where the coefficient C and the exponent n are given as functions of the flume's width. Coefficients and exponents are given in tables found in the *Water Measurement Manual*.

Example 100 – Discharge through a Parshall flume

For a flume of width *5 ft*, and depth, $h_a = 1.2\ ft$, determine the discharge through the flume.

From the *Water Measurement Manual*, $C = 20$ and $n = 1.59$. The resulting discharge is:

$$Q = C\,h_a^{\,n} = 20 \times 1.2^{1.59} = 26.73\ cfs.$$

Other flumes have been designed for special uses, such as for measurement of flows containing large amounts of sediment. See the *Water Measurement Manual* for discussion of special-type flumes.

0370 Hydraulic Modeling

Hydraulic modeling is the practice of testing small-scale hydraulic systems (the model) in the laboratory and translating the favorable results to equivalent quantities of the full-scale system (the prototype). Hydraulic modeling should be performed if sound design and dependable operation of the prototype cannot be accomplished with other recognized methods of analyses. The practice of hydraulic modeling is based on ensuring similarity between model and prototype.

0371 Similarity between Models and Prototypes

The figure below illustrates the concept of prototypes and models. A prototype is the full-scale hydraulic system of interest, and is represented by the spillway on the left-hand side. A model is a small-scale reproduction of the prototype, as illustrated on the right-hand side of the figure.

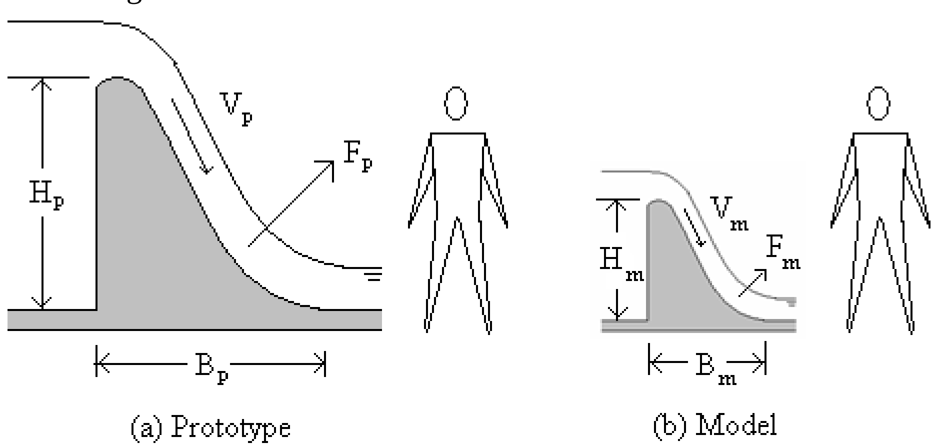


Figure 90. Prototype and model quantities.

The following figure shows the prototype and the model of a spillway flow.

Figure 91. Prototype and model of an ogee spillway (courtesy of the Utah Water Research Laboratory, Utah State University).

By building the model to resemble the shape of the prototype we ensure *geometric similarity*, as illustrated by the photographs of Figure 91. Referring to quantities in the model with the subscript *m* and those in the prototype with the subscript *p*, the ratio of corresponding lengths in model and prototype is referred to as the *length ratio*, L_r:

$$L_r = \frac{L_m}{L_p} \quad \textbf{[Eq. 223]}$$

Since area is calculated as the product of two lengths, the *area ratio* is defined as:

$$A_r = \frac{A_m}{A_p} = L_r^2 \quad \textbf{[Eq. 224]}$$

The *volume ratio* is, consequently, defined as:

$$(Vol)_r = \frac{(Vol)_m}{(Vol)_p} = L_r^3 \quad \textbf{[Eq. 225]}$$

Example 101 – Geometric similarity calculations

A spillway that is 24 ft high ($H_p = 24\ ft$) is to be built at a dam. To check the hydraulic characteristics of the spillway a model with a length ratio $L_r = 1/10$ is to be built in the laboratory. (a) What would be the height of the model, H_m? (b) If the flow cross-section in the model will have an area $A_m = 1.2\ ft^2$, what would be the equivalent area in the prototype? (c) If the volume of water over the prototype is estimated to be $(Vol)_p = 480\ ft^3$, what is the estimated volume of water over the model, $(Vol)_m$?

(a) The model height is:

$$H_m = L_r H_p = (1/10) \times 24\ ft = 2.4\ ft$$

(b) The prototype area is:

$$A_p = A_m/A_r = A_m/L_r^2 = 1.2\ ft^2/(1/10)^2 = 120\ ft^2$$

(c) The model volume is:

$$(Vol)_m = (Vol)_r(Vol)_p = L_r^3(Vol)_p = (1/10)^3 \times 480\ ft^3 = 0.48\ ft^3$$

0372 Hydraulic Modeling of Enclosed Flows (Pipelines)

Enclosed flow models generally require that the Reynolds number, *Re*, of the model equals the *Re* of the prototype i.e. the *Re* ratio equals 1.

Using Reynolds number similarity, the velocity, time, and discharge ratios are:

$$V_r = \frac{\nu_r}{L_r} \qquad \textbf{[Eq. 226]}$$

$$T_r = \frac{L_r}{V_r} = \frac{L_r^2}{\nu_r} \qquad \textbf{[Eq. 227]}$$

$$Q_r = A_r V_r = L_r^2 \cdot \frac{\nu_r}{L_r} = L_r \nu_r \qquad \textbf{[Eq. 228]}$$

Where the kinematic viscosity ratio, ν_r, is the model kinematic viscosity divided by the prototype kinematic viscosity.

Example 102 – Control valve model calculation (pressurized flow)

A model of a control valve for a water pipeline is to be tested in the laboratory. The prototype valve has a diameter $D_p = 4.0\ ft$, while the model will have a diameter $D_m = 0.5\ ft$. If the model is tested using air, determine the velocity, time, and discharge ratios. The kinematic viscosities to use are $\nu_m = \nu_{air} = 1.58 \times 10^{-4}\ ft^2/s$, and $\nu_p = \nu_{water} = 1.22 \times 10^{-5}\ ft^2/s$.

The length ratio is:

$$L_r = D_m/D_p = 0.5\ ft/4.0\ ft = 1/8$$

While the kinematic viscosity ratio is:

$$\nu_r = \nu_m/\nu_p = 1.58\times10^{-4}\ ft^2/s/\ 1.22\times10^{-5}\ ft^2/s = 12.95$$

The velocity, time, and discharge ratios are:

$$V_r = \frac{\nu_r}{L_r} = \frac{12.95}{1/8} = 103.6$$

$$T_r = \frac{L_r^2}{\nu_r} = \frac{(1/8)^2}{12.95} = 1.2\times10^{-3}$$

$$Q_r = L_r \nu_r = (1/8)\times 12.95 = 1.62$$

0373 Hydraulic Modeling in Open-Channel Flow

Open-channel or free-surface models generally require that the Froude number, *Fr*, of the model equals the *Fr* of the prototype i.e. the *Fr* ratio equals 1.

Using Froude number similarity, the velocity, time, and discharge ratios are:

$$V_r = L_r^{1/2} \qquad \textbf{[Eq. 229]}$$

$$T_r = \frac{L_r}{V_r} = \frac{L_r}{L_r^{1/2}} = L_r^{1/2} \qquad \textbf{[Eq. 230]}$$

$$Q_r = A_r V_r = L_r^2 \cdot L_r^{1/2} = L_r^{5/2} \qquad \textbf{[Eq. 231]}$$

Example 103 – Stilling basin model calculation (free-surface flow)

A model of a stilling basin is built at a length ratio $L_r = 1/20$. Using Froude similarity, what are the velocity, time, and discharge ratios? If the length of the prototype basin is designed to be *40 ft*, what is the model dimension? If the discharge of prototype is *2000 cfs*, what is the flow required for the model?

Using Froude number similarity, the velocity, time, and discharge ratios are:

$$V_r = T_r = L_r^{1/2} = \sqrt{1/20} = 0.2236.$$

$$Q_r = A_r V_r = L_r^2 \cdot L_r^{1/2} = L_r^{5/2} = (1/20)^{5/2} = 0.000559$$

Given the prototype basin length $L_p = 40\ ft$, then the model basin length is:

$$L_m = L_r\, L_p = (1/20) \times 40\ ft = 2\ ft$$

Finally, given the prototype discharge $Qp = 2000\ cfs$, then the model discharge is:

$$Q_m = Q_r\, Q_p = 0.000559 \times 2000\ cfs = 1.12\ cfs$$

An important issue in modeling open channel flow is producing the proper scaling of the surface roughness of channel lining. For example, bed roughness in streams and hydraulic structures may be provided by large, stable, rock lining known as riprap. Riprap model studies can be conducted to ensure geometric similarity. However, density differences between the riprap model material and water need to be taken into account if buoyancy effects are to be simulated. Figure 92 illustrates the use of smaller rock (of like density) to simulate the larger rock in a drop structure.

Figure 92. Riprap hydraulic model (source: USDA - ARS)

0374 Limitations of Models

The maximum size of hydraulic models is limited by laboratory physical installations (e.g., space, pumping capacity) and the minimum size of models is limited by the similarity laws. For example, if the model size in an open channel model that follows Froude similarity is too small, viscous effects may become significant. Viscosity may produce undesirable effects in such a model that are not properly scalable to the prototype. Also, if the resulting open-channel model is too shallow, surface tension effects (such as capillarity waves) may appear that would not otherwise be relevant to the prototype performance. In the case of distorted open-channel models (such as for wide, shallow streams), if the vertical model size is much larger than the horizontal size, the resulting flow may include secondary velocity components much larger than the prototype flow. The resulting flow field then would be too distorted to be scaled up to the prototype size. For a detailed discussion on model limitations and similarity laws please refer to Kobus (1980).

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Exhibit 1 – Dimensions and units of measurement

The following table (Table E1.1) shows the dimensions of a variety of physical quantities in terms of the basic units *mass, length, time* (M,L,T) or *force, length, time* (F,L,T). The table also shows the preferred units for those quantities in both the International System (S.I.) and the English System (E.S.) of units. Additional units commonly used for the quantities listed are shown in the last column of the table.

Table E1.1 – Dimensions and units of measurement

	Dimensions		**Preferred units**		**Other**
Quantity	(M,L,T)	(F,L,T)	S.I.	E.S.	**units**
Length (L)	L	L	m	ft	in, mi
Time (T)	T	T	s	s	h, d, min
Mass (M)	M	FT^2L^{-1}	kg	slug	
Area (A)	L^2	L^2	m^2	ft^2	Ac
Volume (Vol)	L^3	L^3	m^3	ft^3	Ac-ft
Velocity (V)	LT^{-1}	LT^{-1}	m/s	ft/s or fps	--
Acceleration (a)	LT^{-2}	LT^{-2}	m/s^2	ft/s^2	--
Discharge (Q)	L^3T^{-1}	L^3T^{-1}	m^3/s	ft^3/s or cfs	--
Kinematic viscosity (ν)	L^2T^{-1}	L^2T^{-1}	m^2/s	ft^2/s	St
Force (F)	MLT^{-2}	F	N	lb	--
Pressure (p)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi, atm
Shear stress (τ)	$ML^{-1}T^{-2}$	FL^{-2}	Pa	lb/ft^2	psi
Density (ρ)	ML^{-3}	FT^2L^{-4}	kg/m^3	$slug/ft^3$	--
Specific weight (ω)	$ML^{-2}T^{-2}$	FL^{-3}	N/m^3	lb/ft^3	--
Energy/Work/Heat (E)	ML^2T^{-2}	FL	J	lb ft	--
Power (P)	ML^2T^{-3}	FLT^{-1}	W	lb ft/s	hp
Dynamic viscosity (μ)	$ML^{-1}T^{-1}$	FTL^{-2}	N s/m^2	lb s/ft^2	P

The units used in Table E1-1 are defined as follows:

Ac	: acre, a unit of area
Ac-ft	: acre × feet
atm	: atmosphere
cfs	: cubic feet per second
fps	: feet per second
ft	: foot or feet
hp	: horse power
in	: inch
J	: joule
kg	: kilogram
lb	: pound
m	: meter
N	: newton
mi	: mile
P	: poise
Pa	: Pascal
psi	: pounds per square inch
s	: second
St	: stokes
W	: watt

Exhibit 2 – Selected conversion factors for units of measurement

The following tables (Tables E2.1 and E2.2) show a number of selected conversion factors for units of measurement in both the English System (E.S.) and the International System (S.I.). The conversion factors are grouped by dimensions. The units are defined as follows:

Ac	: a unit of area
atm	: atmosphere (unit of pressure)
bar	: bar (unit of pressure)
Btu	: British thermal unit (unit of work, energy, or heat)
cal	: calorie (work, energy, or heat)
cm	: centimeter (1/100 of a meter)
cP	: centipoise (1/100 of a poise)
cSt	: centistoke (1/100 of a stoke)
d	: day
dyn	: dyne = g·cm/s^2 (unit of force)
ft	: foot or feet
ftH$_2$0	: feet of water (unit of pressure)
g	: gram (unit of mass)
gal	: gallon (unit of volume)
h	: hour
ha	: hectare (an S.I. unit of area)
hp	: horse power (unit of power)
in	: inch
inH$_2$0	: inches of water (pressure)
inHg	: inches of mercury (pressure)
J	: Joule (work, energy, or heat)
K	: Kelvin (absolute temperature)
kcal	: kilocalorie (1000 calories, heat)
kg	: kilogram (1000 grams, mass)
kip	: kip (1000 lb, force)
km	: kilometer (1000 meters)
kN	: kilonewton (1000 newtons)
kPa	: kilopascal (1000 Pascals)
kph	: kilometers per hour
kW	: kilowatt (1000 watts, power)
kW·h	: kilowatt-hour (work, energy, or heat)
l	: liter (unit of volume)
lb	: pound (unit of force)
m	: meter
mH$_2$O	: meters of water (pressure)
mi	: mile
min	: minute
ml	: milliliter (1/1000 of a liter)
mm	: millimeter (1/1000 of a meter)
mmHg	: millimeters of mercury (press.)
mph	: miles per hour
N	: newton (unit of force)
^{o}	: degree for angular measurement
^{o}C	: Centigrade degree (temp.)
^{o}F	: Fahrenheit degree (temp.)
^{o}R	: Rankine degree (absolute temp)
oz	: ounce (1/16 pound, force)
P	: poise (dynamic viscosity)
Pa	: Pascal (unit of pressure)
psf	: pounds per square foot (lb/ft^2, pressure)
psi	: pounds per square inch (lb/in^2, pressure)
r	: radian (dimensionless, angle)
s	: second
slug	: slug unit of mass = lb·s^2/ft
St	: stokes (kinematic viscosity)
W	: watt (unit of power)
yd	: yard
yr	: year

Table E2.1 – Selected conversion factors for units of measurement – part 1

DIMENSIONS	BASIC CONVERSION FACTORS	
LENGTH	1 ft = 0.3048 m	1 m = 3.2808 ft
MASS	1 slug = 14.5939 kg	1 kg = 0.0685 slug
FORCE	1 lb = 4.4482 N	1 N = 0.2248 lb
TIME	(same basic units)	1 s = 1 s
ABSOLUTE TEMP.	1 ^oR = 5/9 K	1 K = 9/5 ^oR
DIMENSIONS	**OTHER CONVERSION FACTORS**	
LENGTH	1 ft = 0.3048 m 1 ft = 30.48 cm 1 in = 2.54 cm 1 mi = 1.609 km	1 m = 3.2808 ft 1 cm = 0.0328 ft 1 cm = 0.3937 in 1 km = 0.6214 mi
AREA	1 ft^2 = 0.0929 m^2 1 ha = 10^4 m^2 1 ha = 2.4710 Ac 1 in^2 = 6.4516 cm^2	1 m^2 = 10.7639 ft^2 1 Ac = 43560.1742 ft^2 1 Ac = 0.4047 ha 1 cm^2 = 0.1550 in^2
VOLUME	1 ft^3 = 0.0283 m^3 1 in^3 = 16.3871 cm^3 1 ft^3 = 28.3168 l 1 ft^3 = 7.4805 gal 1 l = 1000 cm^3 1 Ac·ft = 43560.1742 ft^3	1 m^3 = 35.3147 ft^3 1 cm^3 = 0.0610 in^3 1 l = 0.0353 ft^3 1 gal = 0.1337 ft^3 1 l = 0.001 m^3 1 ft^3 = 2.2957 10^{-5} Ac·ft
VELOCITY	1 ft/s = 0.3048 m/s 1 mph = 0.4470 m/s 1 mph = 1.6093 kph	1 m/s = 3.2808 ft/s 1 m/s = 2.2369 mph 1 kph = 0.6214 mph
ACCELERATION	acceleration of gravity, g = 32.174 ft/s^2 = 9.8067 m/s^2 1 ft/s^2 = 0.3048 m/s^2	1 m/s^2 = 3.2808 ft/s^2
DISCHARGE	1 ft^3/s = 0.0283 m^3/s 1 ft^3/s = 7.4805 gal/s 1 ft^3/s = 448.8312 gal/min 1 ft^3/s = 28.3168 l/s 1 l/s = 15.8503 gal/min	1 m^3/s = 35.3147 ft^3/s 1 m^3/s = 264.1720 gal/s 1 m^3/s = 15850.3231 gal/m 1 m^3/s = 1000 l/s 1 l/s = 0.001 m^3/s
MASS	1 slug = 14.5939 kg 1 slug = 14593.9029 g 1 g = slug	1 kg = 0.0685 slug 1 kg = 1000 g 1 g = 0.001 kg
FORCE	1 lb = 4.4482 N 1 lb = 16 oz 1 lb = 0.001 kip 1 kip = 1000 lb	1 N = 0.2248 lb 1 N = 10^5 dyn 1 N = 0.0002 kips 1 dyn = 2.2480×10^{-6} lb

Table E2.2 – Selected conversion factors for units of measurement – part 2

DIMENSIONS	**OTHER CONVERSION FACTORS (continued)**	
ENERGY WORK HEAT TORQUE	1 ft-lb = 1.3558 J 1 ft-lb = 0.0013 Btu 1 ft-lb = 0.3238 cal 1 Btu = 778.1693 ft-lb 1 Btu = 251.9958 cal 1 Btu = 1055.0559 J	1 J = 0.7376 ft-lb 1 J = 0.0009 Btu 1 kW·h = 3600000 J 1 kW·h = 3412.1416 Btu 1 J = 10^7 ergs 1 J = 0.2388 cal
POWER	1 ft-lb/s = 1.3558 W 1 hp = 550 ft-lb/s 1 hp = 745.6999 W	1 W = 0.7376 ft-lb/s 1 kW = 737.5621 ft-lb/s
PRESSURE SHEAR STRESS	1 atm = 2116.2166 psf 1 psf = 0.0069 psi 1 psf = 47.8803 Pa 1 mmHg = 0.0193 psi 1 inHg = 0.4912 psi	1 bar = 10^5 Pa = 14.5038 psi 1 kPa = 10^3 Pa = 0.1450 psi 1 ft$H_2$0 = 0.4331 psi 1 in$H_2$0 = 0.0361 psi 1 m$H_2$0 = 0.9102 kPa
DENSITY	1 slug/ft^3 = 515.3788 kg/m^3 1 slug/ft^3 = 0.5154 g/cm^3 1 kg/m^3 = 0.001 g/cm^3	1 kg/m^3 = 0.0019 slug/ft^3 1 g/cm^3 = 1.9403 slug/ft^3 1 g/cm^3 = 1000 kg/m^3
SPECIFIC WEIGHT	1 lb/ft^3 = 157.0875 N/m^3 1 kip/ft^3 = 157087.4638 N/m^3 1 lb/ft^3 = 15.7087 dyn/cm^3	1 N/m^3 = 0.0064 lb/ft^3 1 N/m^3 = 0.1 dyn/cm^3 1 N/m^3 = 6.3658×10^{-6} kip/ft^3
KINEMATIC VISCOSITY	1 ft^2/s = 0.0929 m^2/s 1 ft^2/s = 929.0304 St 1 St = 0.0001m^2/s	1 m^2/s = 10.7639 ft^2/s 1 St = 0.001 ft^2/s 1 m^2/s = 10000 St
DYNAMIC VISCOSITY	1 lb·s/ft^2 = 47.8803 N·s/m^2 1 lb·s/ft^2 = 478.8026 P 1 P = 0.1 N·s/m^2	1 N·s/m^2 = 0.0209 lb·s/ft^2 1 P = 0.0021 lb·s/ft^2 1 N·s/m^2 = 10 P
TEMPERATURE	1 ^{o}R = 5/9 K ^{o}R = ^{o}F + 459.67 ^{o}F = 9/5 ^{o}C + 32	1 K = 9/5 ^{o}R K = ^{o}C + 273.15 ^{o}C = 5/9 (^{o}F - 32)

Exhibit 3 – The Greek alphabet

The following table (Table E3.1) shows the letters of the Greek alphabet in their lower case and upper case forms, their name, and their closest English equivalent.

Table E3.1 – The Greek Alphabet

Lower case	Upper case	Letter name	English equivalent	Lower case	Upper case	Letter name	English equivalent
α	A	Alpha	*a*	ν	N	Nu	*n*
β	B	Beta	*b*	ξ	Ξ	Xi	*x*
γ	Γ	Gamma	*g*	o	O	Omicron	*o*
δ	Δ	Delta	*d*	π	Π	Pi	*p*
ε	E	Epsilon	*e*	ρ	P	Rho	*r*
ζ	Z	Zeta	*z*	σ	Σ	Sigma	*s*
η	H	Eta	*h*	τ	T	Tau	*t*
θ	Θ	Theta	*th*	υ	Y	Upsilon	*u*
ι	I	Iota	*i*	ϕ	Φ	Phi	*ph*
κ	K	Kappa	*k*	χ	X	Chi	*ch*
λ	Λ	Lambda	*l*	ψ	Ψ	Psi	*ps*
μ	M	Mu	*m*	ω	Ω	Omega	*o*

Many of the letters of the Greek alphabet are used as symbols for variables used in hydraulics equations. Some common examples are:

- ρ (rho) Density
- ω (omega) Specific weight (some references use γ for specific weight)
- μ (mu) Dynamic viscosity
- ν (nu) Kinematic viscosity
- τ (tau) Shear stress
- ε (epsilon) Absolute roughness in pipes
- π (pi) Ratio of length of circle to its diameter, constant value: $\pi \approx 3.1416$

In reference to angular measurements, the letters α (alpha), β (beta), γ (gamma), and θ (theta) are commonly used.

The letter Δ (upper-case delta) is typically used to indicate an increment of a quantity, thus, Δh may represent, for example, an increment on a depth h.

The letter ε (epsilon) is typically used to indicate a very small quantity in solution of equations.

Exhibit 4 – Physical properties of water

The following tables (Table E4.1 and E4.2) show values of the following physical properties of water at different temperatures:

- Density
- Specific weight
- Kinematic viscosity
- Dynamic viscosity
- Vapor pressure
- Bulk modulus of elasticity

Table E4.1 shows the properties of water in units of the English System (E.S.), while Table E4.2 shows the same properties in units of the International System (S.I.).

Table E4.1 – Physical properties of water in E.S. units

Temp. **°F**	Density ρ **slug/ft³**	Specific weight, ω **lb/ft³**	Kinematic viscosity, ν **ft²/s**	Dynamic viscosity, μ **lb·s/ft²**	Vapor pressure, p_v **psi-abs**	Bulk Modulus of Elasticity, E **psi**
32	1.940	62.42	1.931×10^{-5}	3.746×10^{-5}	0.09	2.93×10^{5}
40	1.940	62.43	1.664×10^{-5}	3.229×10^{-5}	0.12	2.94×10^{5}
50	1.940	62.41	1.410×10^{-5}	2.735×10^{-5}	0.18	3.05×10^{5}
60	1.938	62.37	1.217×10^{-5}	2.359×10^{-5}	0.26	3.11×10^{5}
70	1.936	62.30	1.059×10^{-5}	2.050×10^{-5}	0.36	3.20×10^{5}
80	1.934	62.22	9.300×10^{-6}	1.799×10^{-5}	0.51	3.22×10^{5}
90	1.931	62.11	8.260×10^{-6}	1.595×10^{-5}	0.70	3.23×10^{5}
100	1.927	62.00	7.390×10^{-6}	1.424×10^{-5}	0.95	3.27×10^{5}
110	1.923	61.86	6.670×10^{-6}	1.284×10^{-5}	1.27	3.31×10^{5}
120	1.918	61.71	6.090×10^{-6}	1.168×10^{-5}	1.69	3.33×10^{5}
130	1.913	61.55	5.580×10^{-6}	1.069×10^{-5}	2.22	3.34×10^{5}
140	1.908	61.38	5.140×10^{-6}	9.810×10^{-6}	2.89	3.30×10^{5}
150	1.902	61.20	4.760×10^{-6}	9.050×10^{-6}	3.72	3.28×10^{5}
160	1.896	61.00	4.420×10^{-6}	8.380×10^{-6}	4.74	3.26×10^{5}
170	1.890	60.80	4.130×10^{-6}	7.800×10^{-6}	5.99	3.22×10^{5}
180	1.883	60.58	3.850×10^{-6}	7.260×10^{-6}	7.51	3.18×10^{5}
190	1.876	60.36	3.620×10^{-6}	6.780×10^{-6}	9.34	3.13×10^{5}
200	1.868	60.12	3.410×10^{-6}	6.370×10^{-6}	11.52	3.08×10^{5}
212	1.860	59.83	3.190×10^{-6}	5.930×10^{-6}	14.70	3.00×10^{5}

Table E4.2 – Physical properties of water in S.I. units

Temp. ^oC	Density ρ kg/m^3	Specific weight, ω kN/m^3	Kinematic viscosity, ν m^2/s	Dynamic viscosity, μ $N\cdot s/m^2$	Vapor pressure, p_v kPa-abs	Bulk Modulus of Elasticity, E kPa
0	999.8	9.805	1.785×10^{-6}	1.781×10^{-3}	0.61	2.02×10^6
5	1000.0	9.807	1.519×10^{-6}	1.518×10^{-3}	0.87	2.06×10^6
10	999.7	9.804	1.306×10^{-6}	1.307×10^{-3}	1.23	2.10×10^6
15	999.1	9.798	1.139×10^{-6}	1.139×10^{-3}	1.70	2.15×10^6
20	998.1	9.789	1.000×10^{-6}	1.002×10^{-3}	2.34	2.18×10^6
25	997.0	9.777	8.930×10^{-7}	8.900×10^{-4}	3.17	2.22×10^6
30	995.7	9.764	8.000×10^{-7}	7.980×10^{-4}	4.24	2.25×10^6
35	994.0	9.747	7.290×10^{-7}	7.300×10^{-4}	5.81	2.27×10^6
40	992.2	9.730	6.580×10^{-7}	6.530×10^{-4}	7.38	2.28×10^6
45	990.1	9.710	6.055×10^{-7}	6.000×10^{-4}	9.86	2.29×10^6
50	988.0	9.689	5.530×10^{-7}	5.530×10^{-4}	12.33	2.29×10^6
55	985.6	9.666	5.135×10^{-7}	5.135×10^{-4}	16.13	2.29×10^6
60	983.2	9.642	4.740×10^{-7}	4.740×10^{-4}	19.92	2.28×10^6
65	980.5	9.620	4.435×10^{-7}	4.435×10^{-4}	25.54	2.27×10^6
70	977.8	9.589	4.130×10^{-7}	4.130×10^{-4}	31.16	2.25×10^6
75	974.8	9.560	3.885×10^{-7}	3.885×10^{-4}	39.25	2.23×10^6
80	971.8	9.530	3.640×10^{-7}	3.640×10^{-4}	47.34	2.20×10^6
85	968.6	9.498	3.395×10^{-7}	3.395×10^{-4}	58.75	2.17×10^6
90	965.3	9.466	3.150×10^{-7}	3.150×10^{-4}	70.10	2.14×10^6
95	961.9	9.433	3.045×10^{-7}	2.990×10^{-4}	85.72	2.11×10^6
100	958.4	9.399	2.940×10^{-7}	2.820×10^{-4}	101.33	2.07×10^6

Exhibit 5 – Variation of atmospheric pressure with elevation

The following table shows the variation of atmospheric pressure with elevation.

English System		International System	
Elevation above sea level	Atmospheric pressure p_{atm}	Elevation above sea level	Atmospheric pressure p_{atm}
(ft)	(psia)	(m)	(kPa)
0	14.69	0	101.32
1000	14.18	300	97.78
2000	13.67	600	94.34
3000	13.18	900	90.98
4000	12.70	1200	87.73
5000	12.23	1500	84.56
6000	11.78	1800	81.49
7000	11.34	2100	78.51
8000	10.92	2400	75.62
9000	10.50	2700	72.83
10000	10.10	3000	70.12

Based on the ICAO (*) Standard Atmosphere
(*) International Civil Aviation Organization

The tables are based on the following data fitting of the *atmospheric pressure*-vs-*elevation* data:

- Units of the English System: p_{atm} (*psia*) = atmospheric pressure, z (*ft*) = elevation

$$p_{atm} = 14.69 - 0.000525 \cdot z + 6.563 \times 10^{-9} \cdot z^2$$

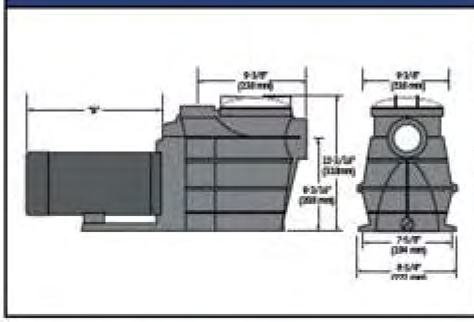
- Units of the International System: p_{atm} (*kPa*) = atmospheric pressure, z (*m*) = elevation

$$p_{atm} = 101.32 - 0.01195 \cdot z + 5.172 \times 10^{-7} \cdot z^2$$

Exhibit 6 - Pipe-system analysis for pump selection

This example illustrates the use of manufacturer-provided pump curves in pump selection. The following figure shows the pump curves provided by a manufacturer for a number of pumps whose sizes are specified in the Figure E6.1.

OVERALL DIMENSIONS



MODEL	Motor Power		Pipe Size	Dimension "A"	
	HP	KW	inches	inches	mm
SP3005X7AZ	3/4	0.55	1 1/2	10 5/8	270
SP3007X10AZ	1	0.75	1 1/2	11	280
SP3010X15AZ*	1 1/2	1.10	2	12 1/8	308
SP3015X20AZ*	2	1.55	2	13 1/16	332
SP3020X25AZ*	2 1/2	1.88	2	13 1/16	332
SP3025X30AZ	3	2.20	2	14 13/64	361

*Super II Pumps available with dual-speed motors.

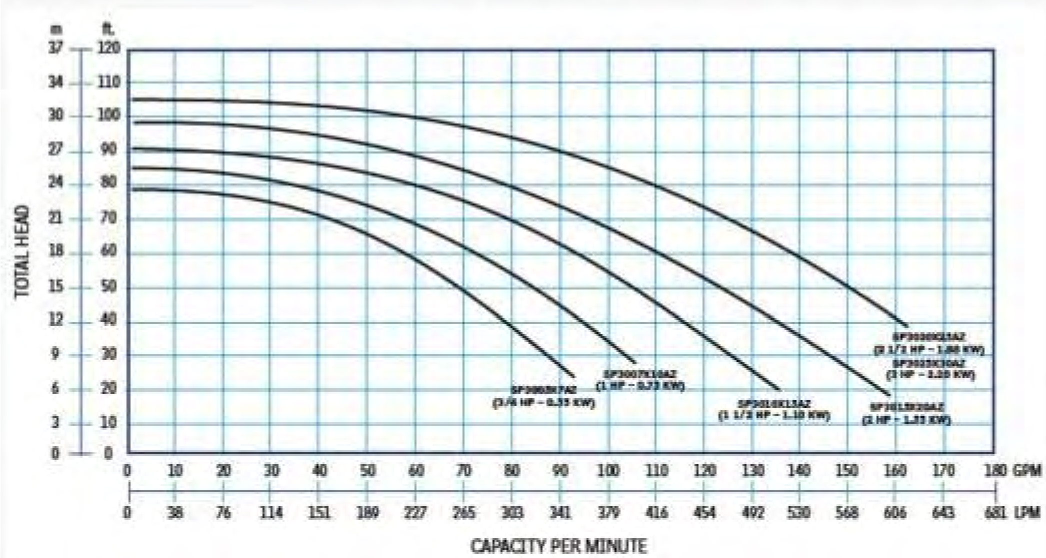


Figure E6.1. Manufacturer-provided pump curves and pump sizes

Suppose that the pump is to be installed in a system as that shown in the following figure. The parameters of the system are the following: $L = 1000\ ft$, $D = 2\ in$, $e = 0.000005\ ft$, $g = 32.2\ ft/s^2$, $\nu = 1.20\times10^{-5}\ ft^2/s$, $K_e = 0.5$, $K_d = 1.0$, (i.e., $\Sigma K = 1.5$), and $\Delta z = 20\ ft$.

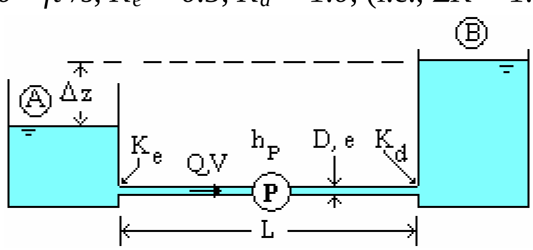


Figure E6-2. Pump-pipeline system for analysis.

A numerical spreadsheet application was used to develop the system curve, which is the capacity and head needed for various operating conditions. The tabular data for this example's system curve are shown below:

System h_p (ft)	Q (gpm)
21	4
30	22
55	45
93	67
142	90
275	135
451	180

The next step is to plot the system curve values of h_p (*ft*) and *Q(gpm)* over the manufacturer's pump curves, as shown in the figure below:

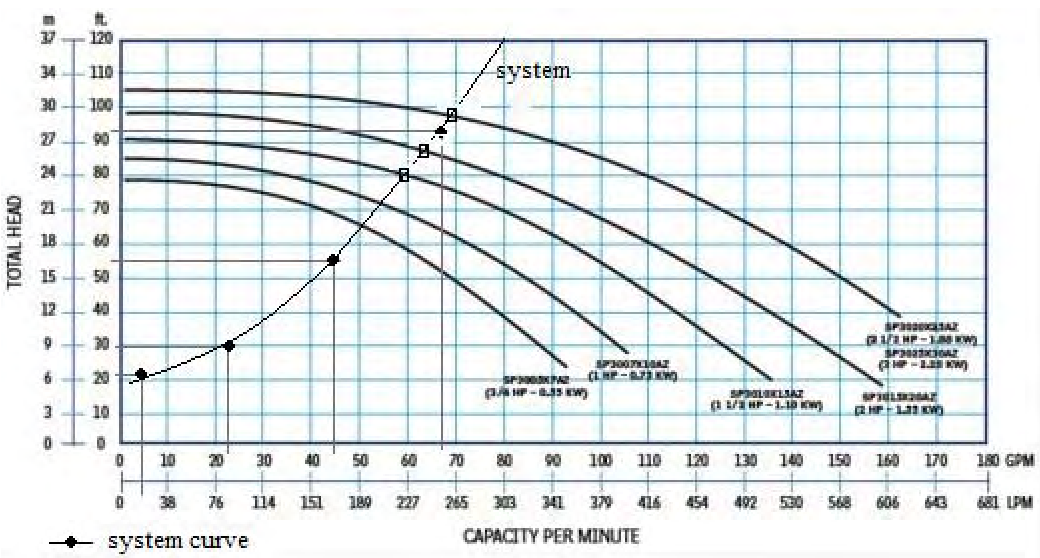


Figure E6-3. System curve plotted against pump curves

The points of intersection of the system curve with the pump curves are the operating points. The three system operating points for the pumps using 2-inch diameter pipe are:

Pump SP3010X15AZ,	*Q = 59 gpm*, $h_P = 80$ *ft*
Pump SP3015X20AZ,	*Q = 64 gpm*, $h_P = 88$ *ft*
Pumps SP3020X25AZ and SP3025X30AZ,	*Q = 69 gpm*, $h_P = 99$ *ft*

Generally, a pump should be selected with an operating point at 80 to 85% of the shut-off head (maximum head).

Exhibit 7 – Culvert flow solutions using nomograms

This exhibit includes nomograms for the calculation of culvert flow for different pipe materials. A nomogram is a graphical device used for solving equations. In a nomogram, the user pinpoints the location of two points in the scales provided and traces a straight line to a third scale. The intersection of the straight line with the third scale is the value of the unknown variable sought. Some nomograms include additional scales corresponding to different conditions for the problem, as well as turning lines to relate the solution to a third variable.

The four nomograms included in this exhibit are the same as charts in *FHWA-NHI-01-020, HYDRAULIC DESIGN OF HIGHWAY CULVERTS, Hydraulic Design Series No. 5*, September 2001. *FHWA, HDS 5* contains 55 additional charts for hydraulic analyses of culverts of different types and flow conditions. Examples are included for each of the nomograms in this exhibit.

The culvert flow types for the four nomograms are illustrated in Figure 64, which is repeated below:

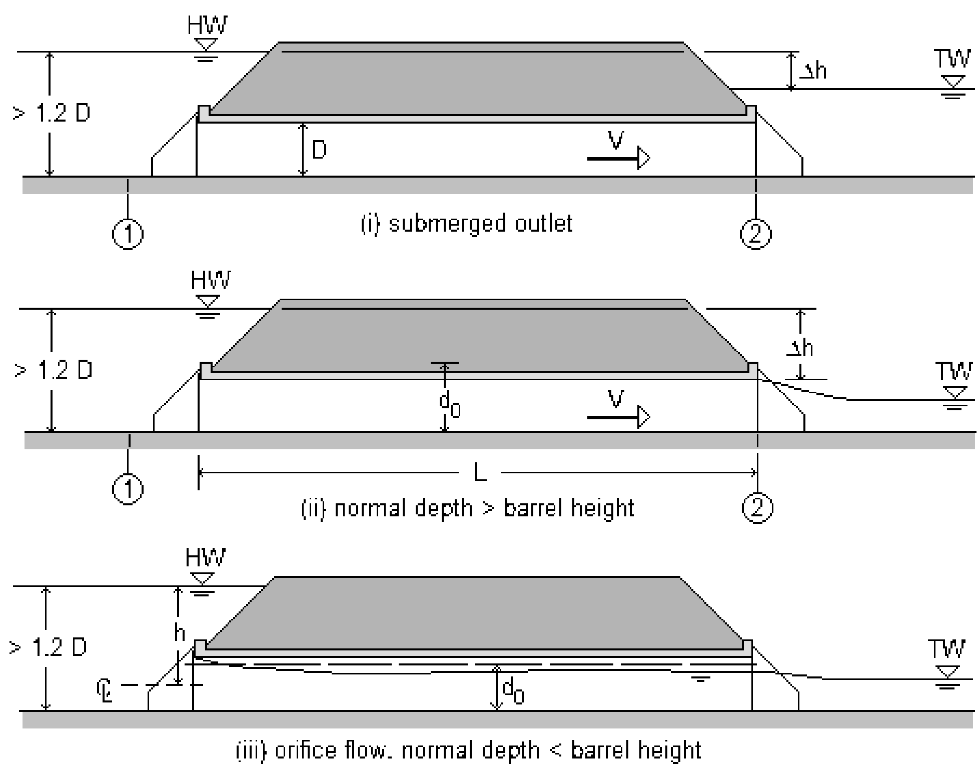


Figure 64. Flow regimes for submerged inlet flow in culverts [repeated].

The four nomograms for this exhibit are described as follows:

- Figure E7-1. Nomogram for headwater depth for concrete pipe culverts with inlet control.
- Figure E7-2. Nomogram for headwater depth for corrugated metal (CM) pipe culverts with inlet control.
- Figure E7-3. Nomogram for concrete pipe culverts flowing full with outlet control, Manning's $n = 0.012$.
- Figure E7-4. Nomogram for head for corrugated metal (CM) pipe culverts flowing full with outlet control, $n = 0.024$.

The variables referenced in these nomograms are described as follows:

- D = culvert pipe diameter (in)
- $\frac{HW}{D}$ = Head water depth to culvert pipe diameter ratio (dimensionless)
- Q = discharge (cfs)
- L = culvert length (ft)
- H or Δh = head (ft), the difference in elevation between the HW depth and downstream depth in a culvert

Notice that variable Δh in Figure 64 is the same as variable H in the nomograms of Figures E7-3 and E7-4 in this exhibit.

Example E7-1. Concrete pipe culvert with inlet control
Consider a concrete pipe culvert with diameter $D = 42\ in = 3.5\ ft$, carrying a discharge $Q = 120\ cfs$. Determine the required head water depth (HW) for the following conditions:

(1) Square edge with headwall; (2) Groove end with headwall; (3) Groove end projecting

Using Figure E7-1, first locate $D = 42\ in$ on scale (4), then $Q = 120\ cfs$ on scale (3), and trace a straight line through these two points and extend it up to scale (1). The point of intersection of the EXAMPLE line with scale (1) corresponds to the value of HW/D = 2.5 for a square edge with headwall. To find the values of HW/D for the other cases (groove end with headwall and groove end projecting), draw a horizontal line from the point found in scale (1) to intersect scale (2) and scale (3), respectively. The values read in scales (2) and (3) are $HW/D = 2.1$ and $HW/D = 2.2$, respectively. The value of the corresponding headwater can be calculated multiplying the HW/D value by the pipe diameter D. Thus, for the three cases required in this example, the following results are obtained:

(1) Square edge with headwall, $HW/D = 2.5$, $D = 2.5 \times 3.5\ ft = 8.75\ ft \approx 8.8\ ft$
(2) Groove end with headwall, $HW/D = 2.1$, $D = 2.1 \times 3.5\ ft = 7.35\ ft \approx 7.4\ ft$
(3) Groove end projecting, $HW/D = 2.2$, $D = 2.2 \times 3.5\ ft = 6.6\ ft$

The nomographs were computed assuming 2% culvert slopes, and are considered to be accurate within 10% for determining the required inlet control headwater.

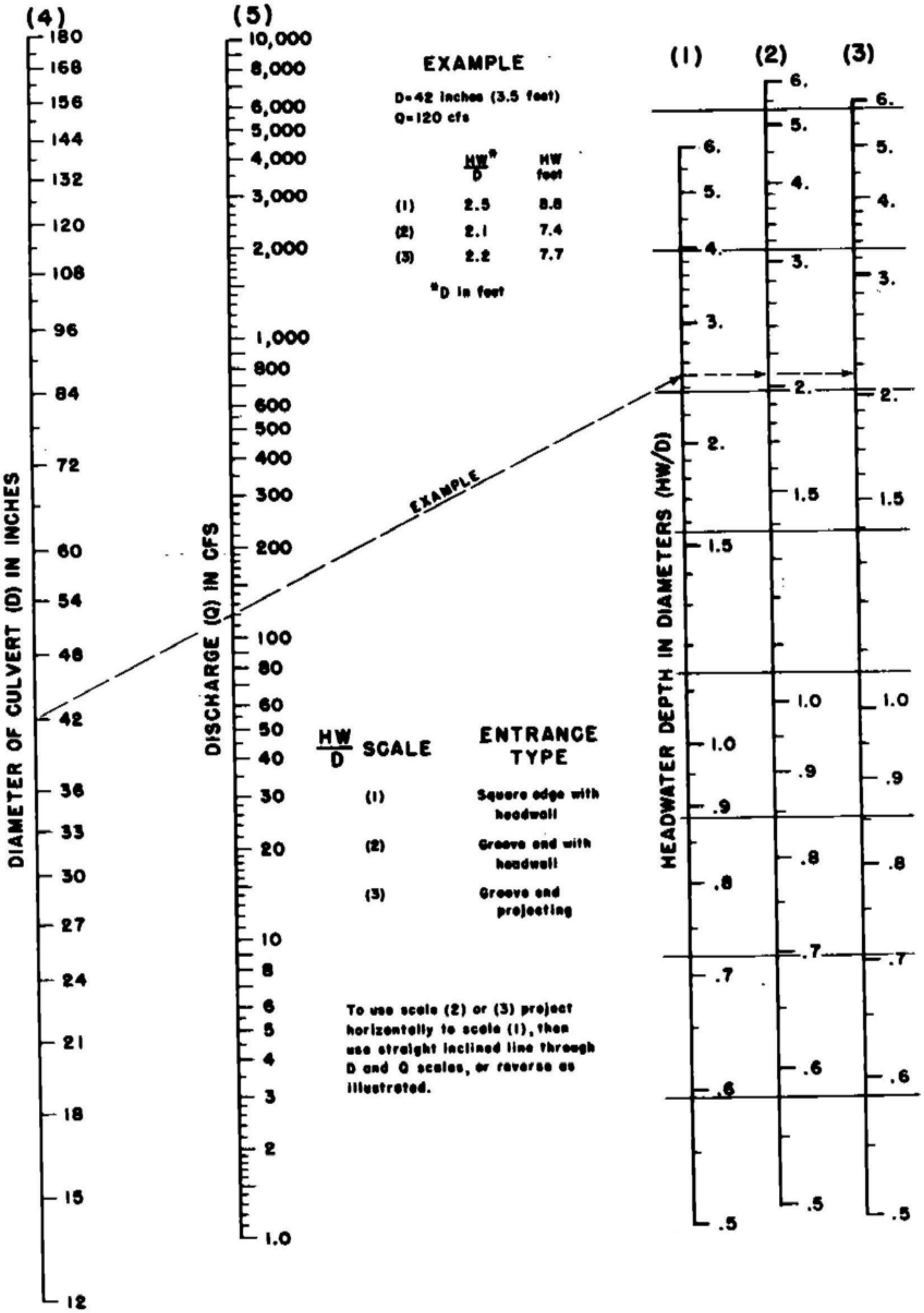


Figure E7-1. Nomogram for headwater depth for concrete pipe culverts with inlet control.

Example E7-2. Corrugated metal (CM) pipe culvert with inlet control
Consider a concrete pipe culvert with diameter $D = 36\ in = 3\ ft$, carrying a discharge $Q = 66\ cfs$. Determine the required head water depth (HW) for the following conditions:

(1) Headwall; (2) Mitered to conform to slope; (3) Projecting

Using Figure E7-2, first locate $D = 36\ in$ on scale (4), then $Q = 66\ cfs$ on scale (3), and trace a straight line through these two points and extend it up to scale (1). The point of intersection of the EXAMPLE line with scale (1) corresponds to the value of $HW/D = 1.8$ for a square edge with headwall. To find the values of HW/D for the other cases (mitered to conform to slope and projecting), draw a horizontal line from the point found in scale (1) to intersect scale (2) and scale (3), respectively. The values read in scales (2) and (3) are $HW/D = 2.1$ and $HW/D = 2.2$, respectively. The value of the corresponding headwater can be calculated multiplying the HW/D value by the pipe diameter D. Thus, for the three cases required in this example, the following results are obtained:

(1) Headwall, $HW/D = 1.8, D = 1.8 \times 3\ ft = 5.4\ ft$
(2) Mitered to conform to slope, $HW/D = 2.1, D = 2.1 \times 3\ ft = 6.3\ ft$
(3) Projecting, $HW/D = 2.2, D = 2.2 \times 3\ ft = 6.6\ ft$

Example E7-3. Concrete pipe culvert flowing full with outlet control ($n = 0.012$)
Determine the diameter of a concrete pipe culvert flowing full with a length $L = 110\ ft$ with an entrance loss coefficient $k_e = 0.5$ if it is to carry a flow $Q = 70\ cfs$ with a head loss $H = 0.94\ ft$.

Using Figure E7-3, first, trace a straight line from the DISCHARGE scale at point $Q = 70\ cfs$ to the HEAD scale at $H = 0.94$. Next, locate the point $L = 110\ ft$ on the LENGTH scale corresponding to $k_e = 0.5$, and trace a straight line through the intersection of the first line with the TURNING LINE, extending it all the way to the DIAMETER scale. The ending point of this second line represents the predicted value for the diameter. In this case, $D = 48\ in = 4\ ft$. Since this value is a standard value for pipe diameters, this will be the design diameter. On the other hand, if the diameter had been, say, $D = 51\ in$, then, it is recommended to use the next larger standard diameter, $D = 54\ in = 4.5\ ft$.

Example E7-4. Corrugated metal (CM) pipe culvert flowing full with outlet control ($n = 0.024$)
Determine the head loss H for a CM culvert with diameter $D = 27\ in$ and length $L = 120\ ft$ if it is to carry a discharge $Q = 35\ cfs$. The entrance loss coefficient is $k_e = 0.9$.

Using Figure E7-4, first, trace a straight line from the DIAMETER scale at $D = 27\ in$ to the point $L = 120\ ft$ in the LENGTH scale corresponding to $k_e = 0.9$. Next, locate the point $Q = 35\ cfs$ in the DISCHARGE scale, and trace a straight line through the intersection of the first line with the TURNING LINE, extending all the way to the HEAD scale. The end of this line will show the required value of, $H = 0.75\ ft$.

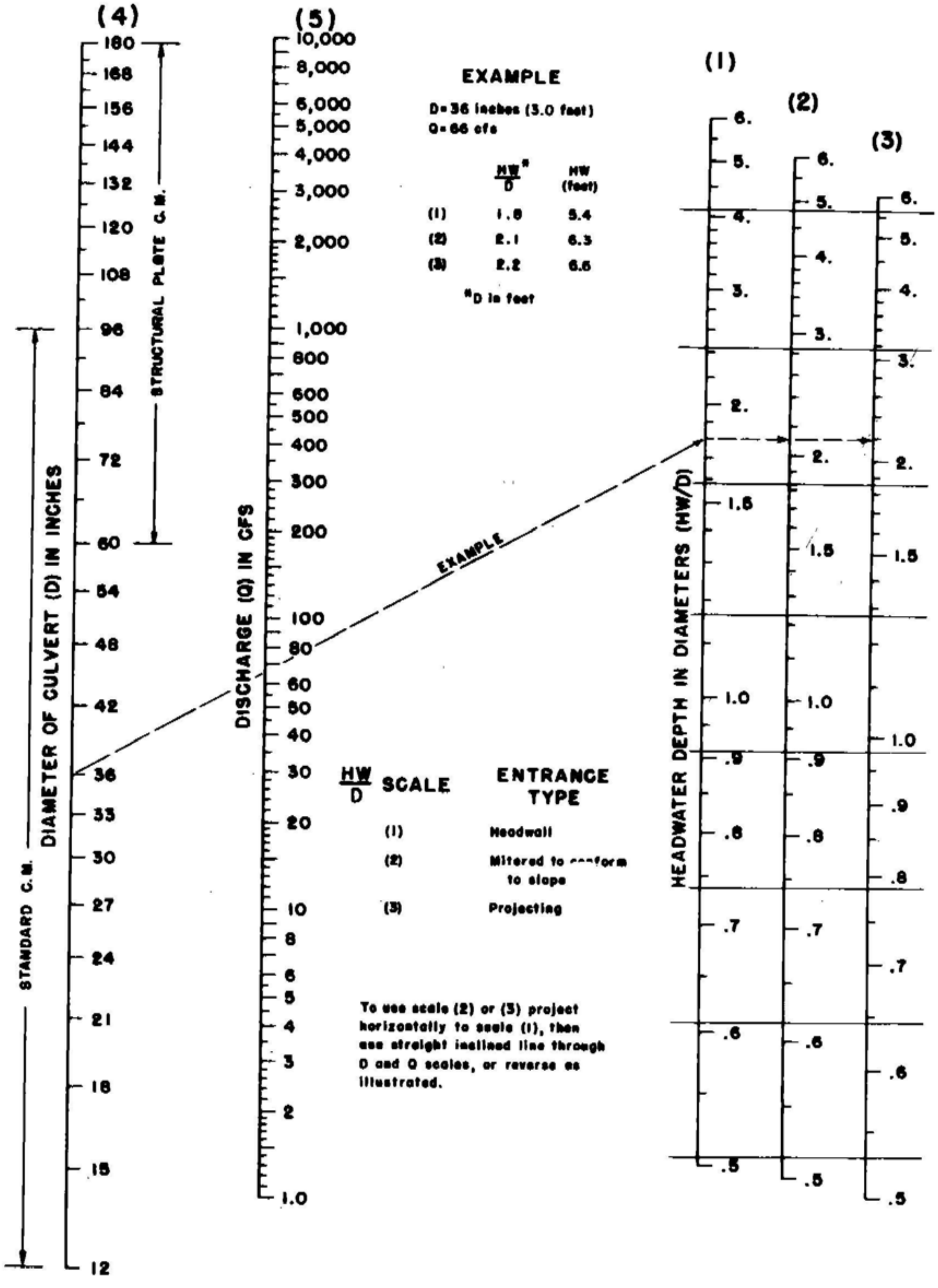


Figure E7-2. Nomogram for headwater depth for corrugated metal (CM) pipe culverts with inlet control.

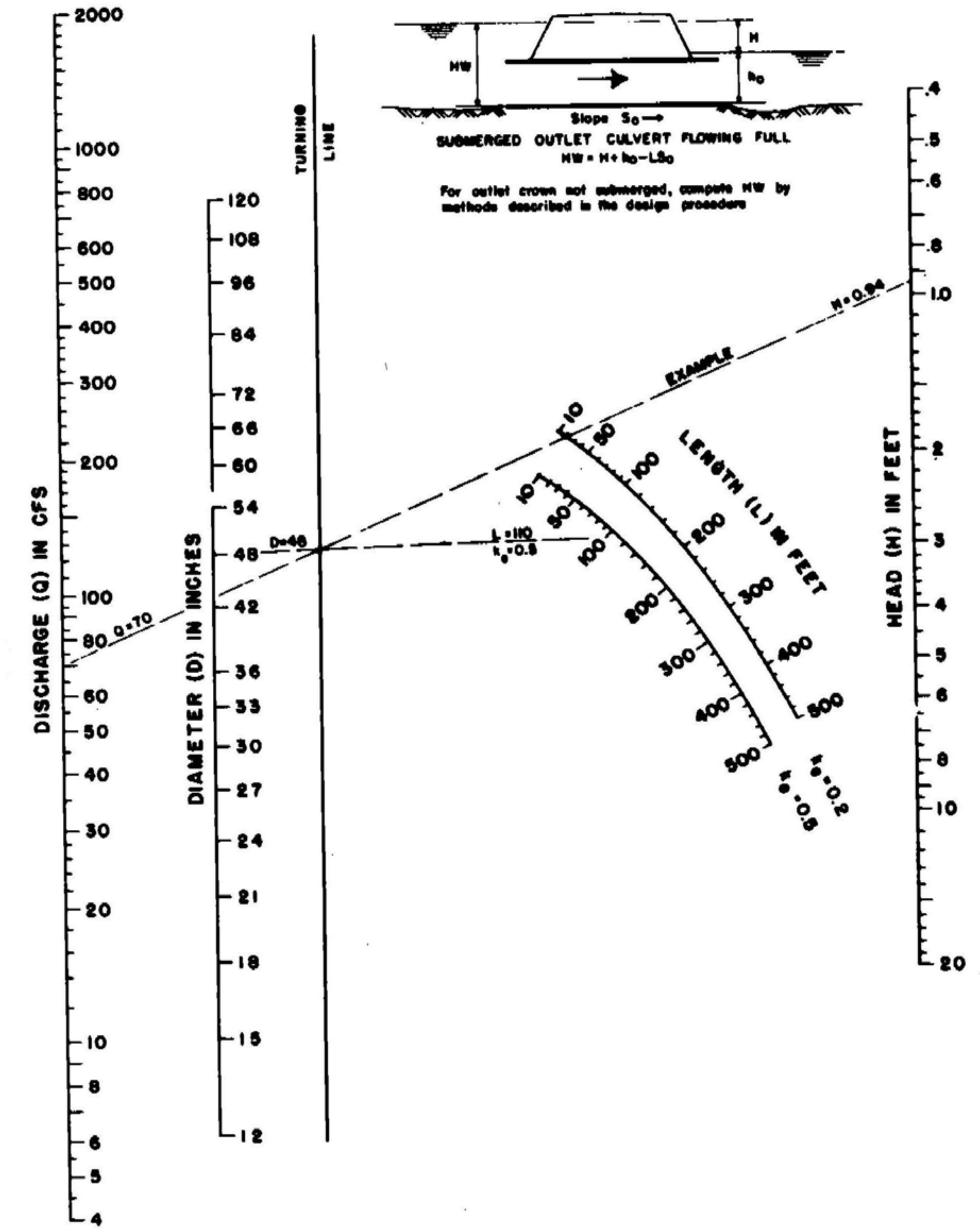


Figure E7-3. Nomogram for concrete pipe culverts flowing full with outlet control, Manning's $n = 0.012$.

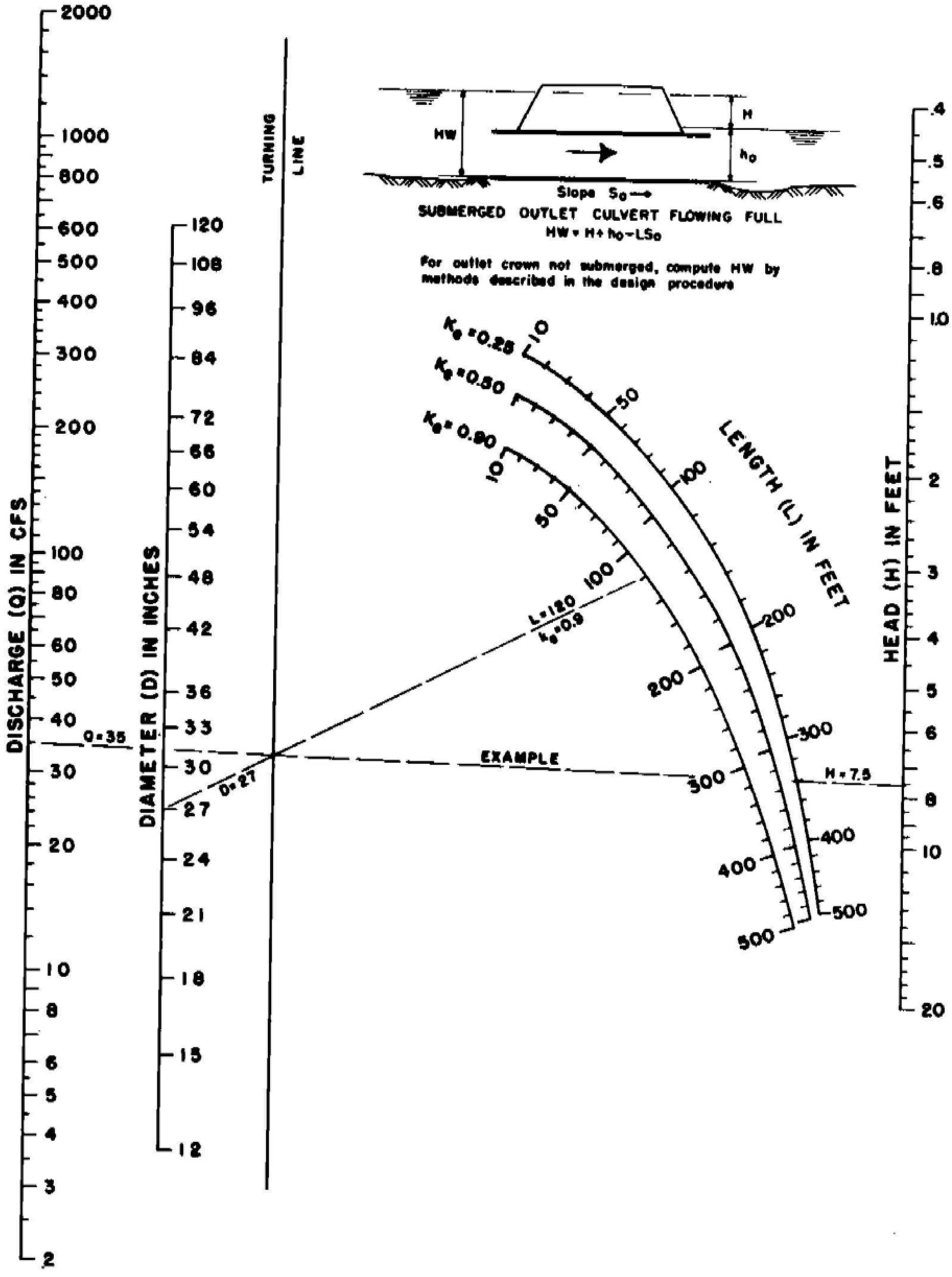


Figure E7-4. Nomogram for head for corrugated metal (CM) pipe culverts flowing full with outlet control, *n = 0.024*.